Exploiting Strict Constraints in the Cylindrical Algebraic Covering

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RWTH Aachen University

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Quantifier-free Non-linear Real Arithmetic (QFNRA)

$p_1 = -x_1^2 - x_2 + 1$

$p_2 = x_1^2 - x_2 - 1$

$p_3 = (x_1 - 0.5)^2 + (x_2 + 1.5)^2 - 0.25$

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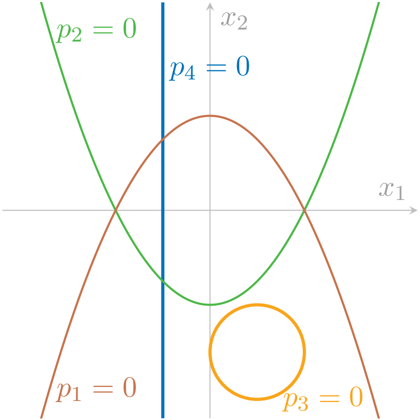

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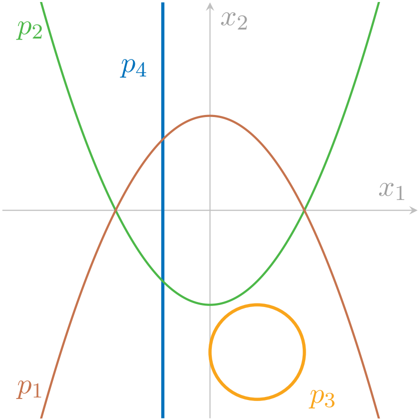

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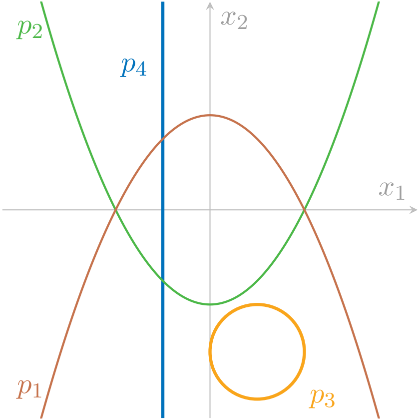

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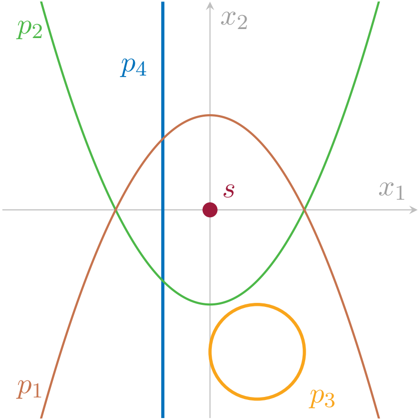
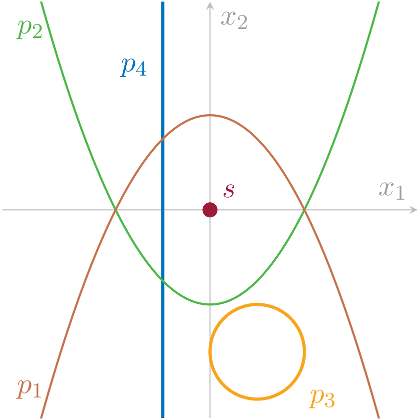


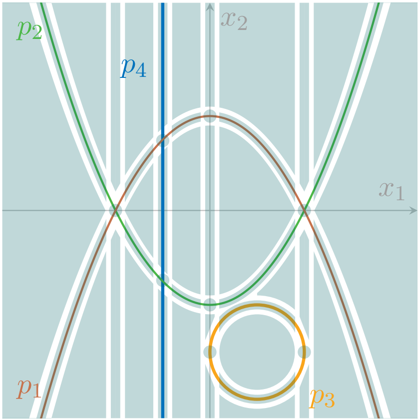
Quantifier-free Non-linear Real Arithmetic (QFNRA)

$$\varphi \quad := \quad 1 < 0 \quad \wedge \quad -1 > 0 \quad \wedge \quad 2.25 \geq 0 \quad \wedge \quad 0.5 \geq 0$$



Cylindrical Algebraic Decomposition (CAD) [Col75]

$$\varphi \quad := \quad p_1 < 0 \quad \land \quad p_2 > 0 \quad \land \quad p_3 \geq 0 \quad \land \quad p_4 \geq 0$$



sign-invariant cell

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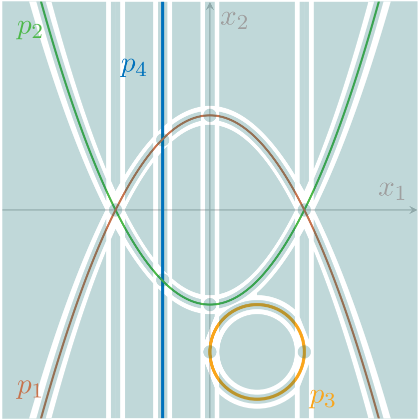

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Sign-invariance $\Rightarrow$ Truth-invariance

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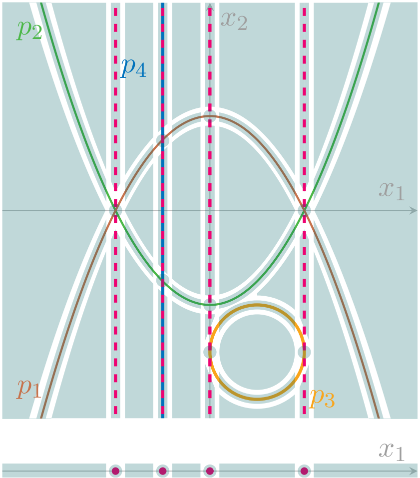

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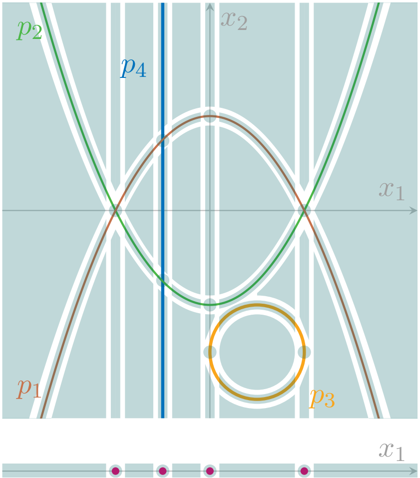

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Sign-invariance ⇒ Truth-invariance

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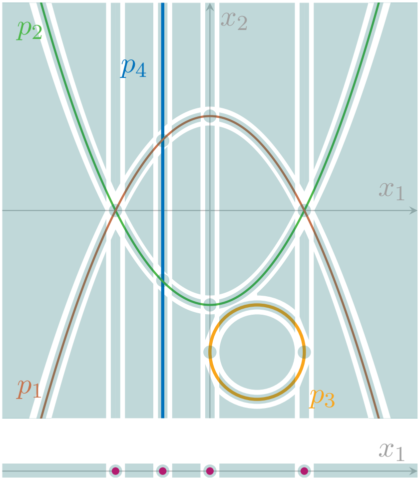

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$\rightarrow \varphi$ is SAT.

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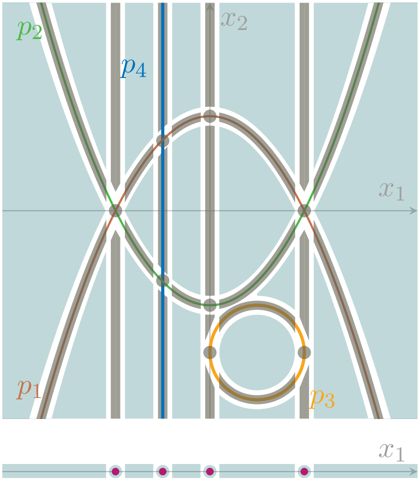

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$p \sim 0$ strict $\Leftrightarrow \sim \in \{<, \neq, >\}$

Cylindrical Algebraic Covering (CAIC) [ÁDEK21]

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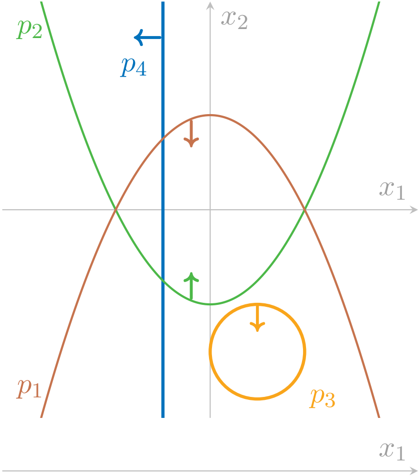

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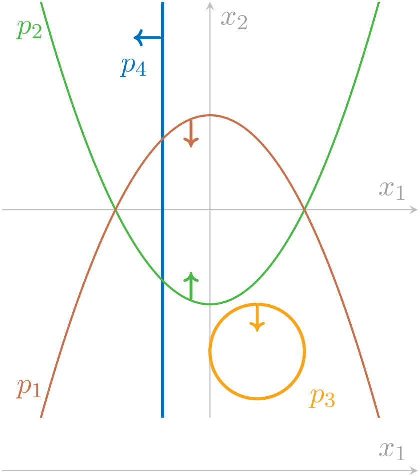

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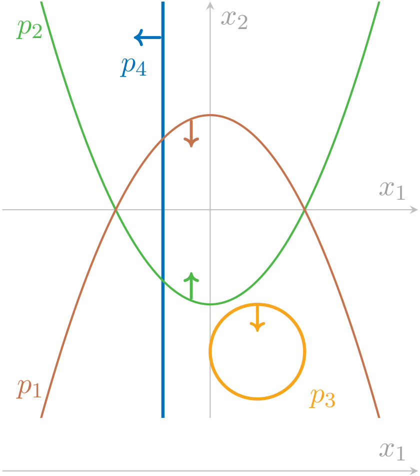

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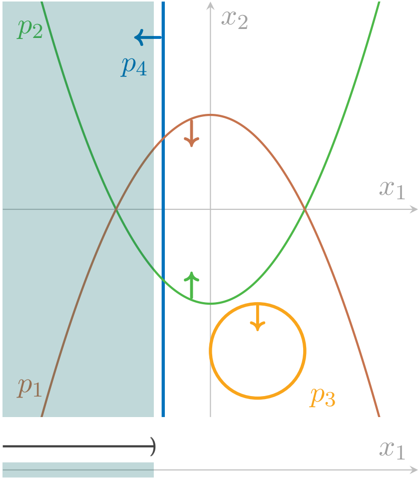

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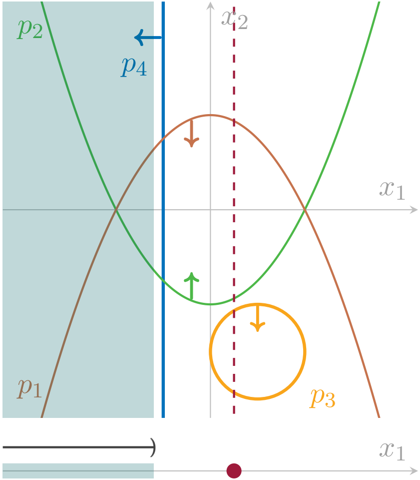

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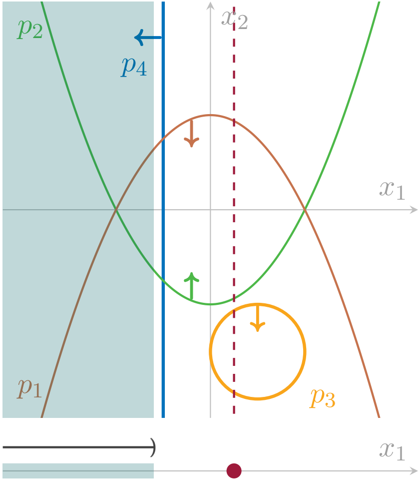

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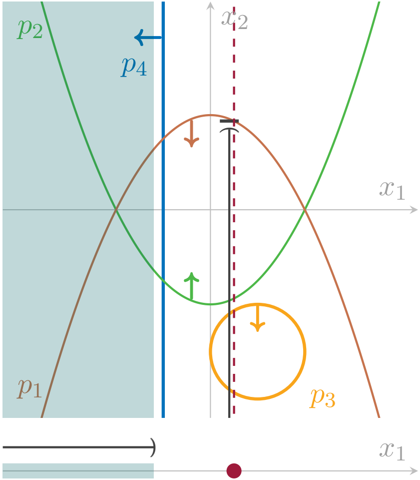

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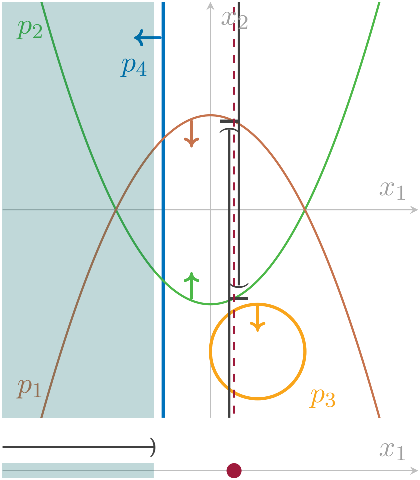

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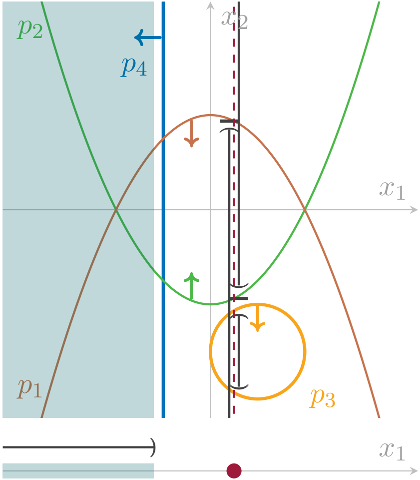

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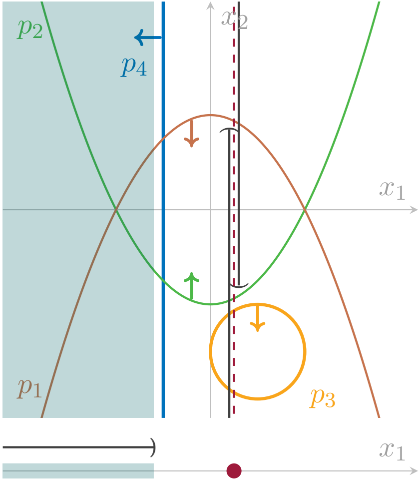

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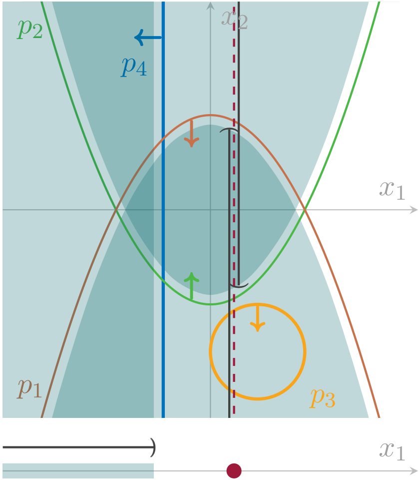

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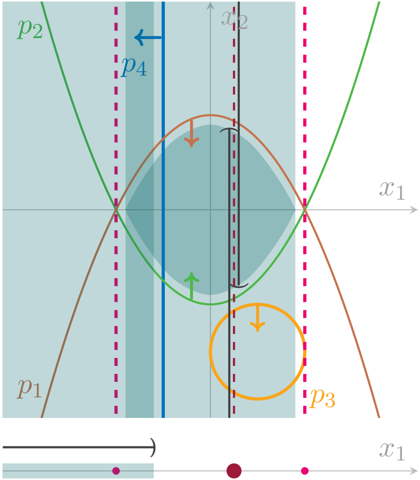

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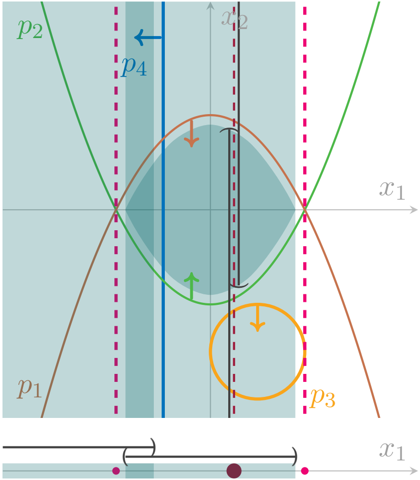

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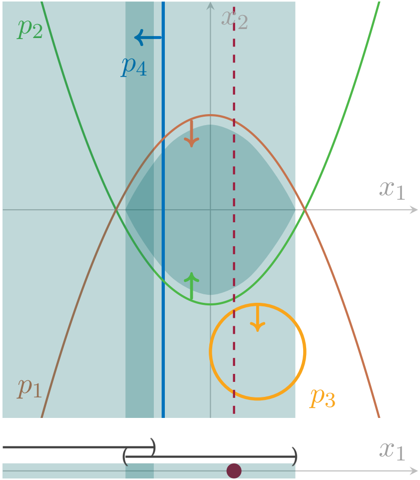

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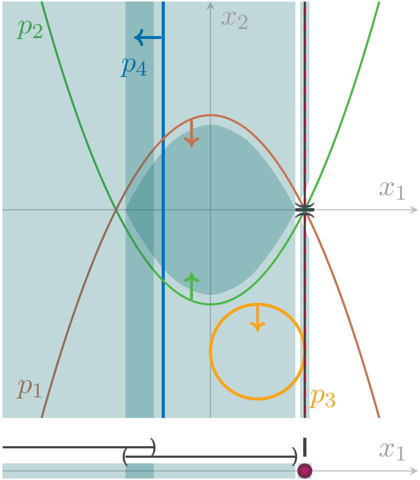

Sample $s = (x_1 : 1)$

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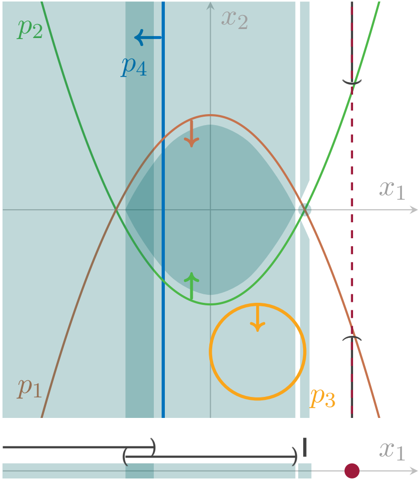

Sample $s = (x_1 : 1.5)$

$p_1(s_1, x_2) = -x_2 - 1.25 \quad \leftarrow$

$p_2(s_1, x_2) = -x_2 + 1.25 \quad \leftarrow$

$p_3(s_1, x_2) = (x_2 + 1.5)^2 + 0.75 \quad \leftarrow$

$p_4(s_1, x_2) = 2$

Cylindrical Algebraic Covering (CAlC) [ÁDEK21]

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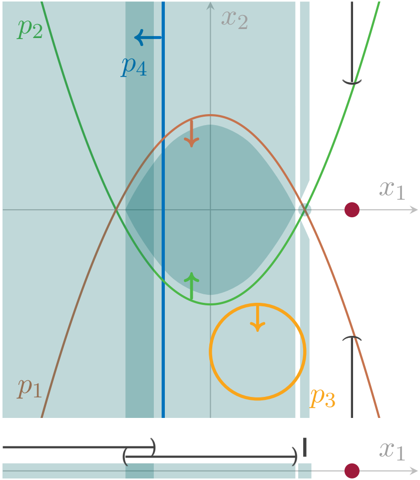

Sample $s = (x_1 : 1.5, x_2 : 0)$

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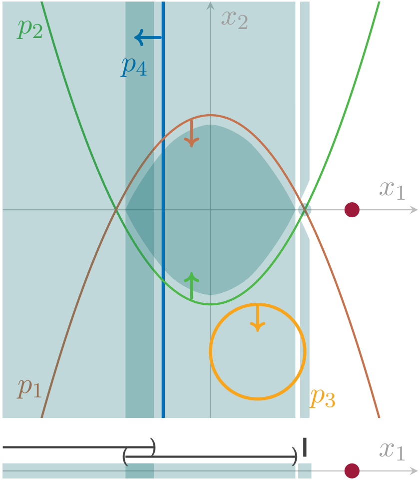

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$\rightarrow \varphi$ is SAT.

Exploiting Strict Constraints

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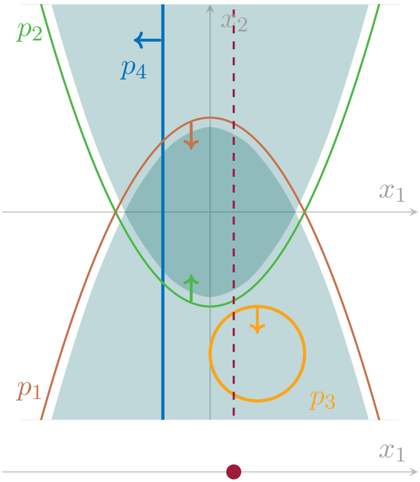

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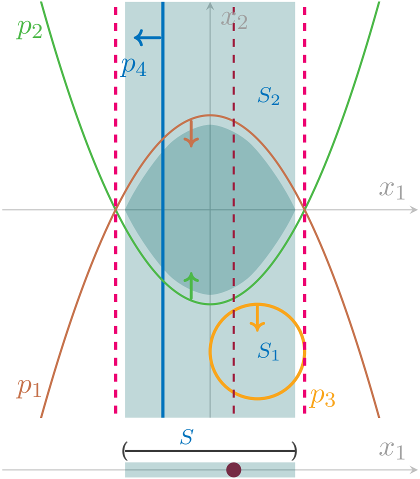

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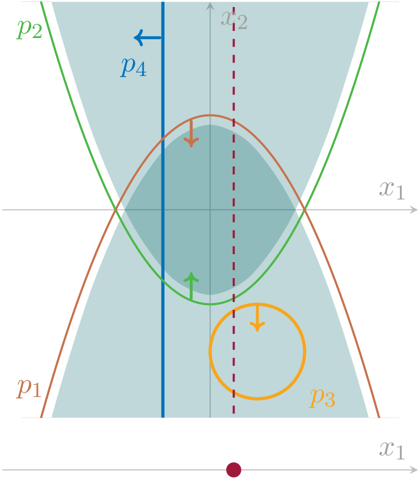

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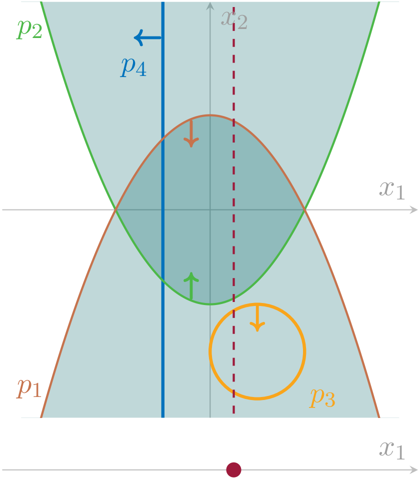
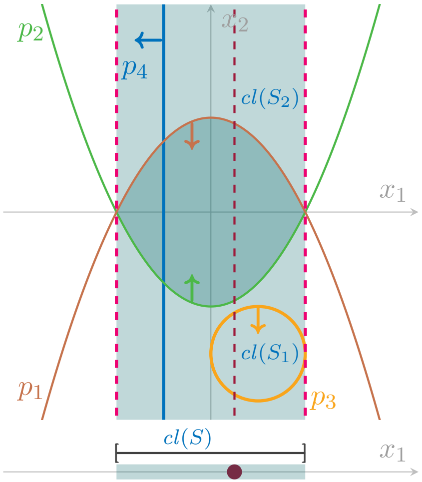


Sample $s = (x_1 : 0.25)$

Exploiting Strict Constraints

$$\varphi := p_1 < 0 \wedge p_2 > 0 \wedge p_3 \geq 0 \wedge p_4 \geq 0$$



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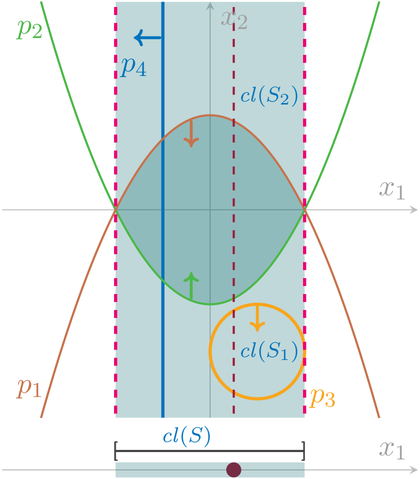


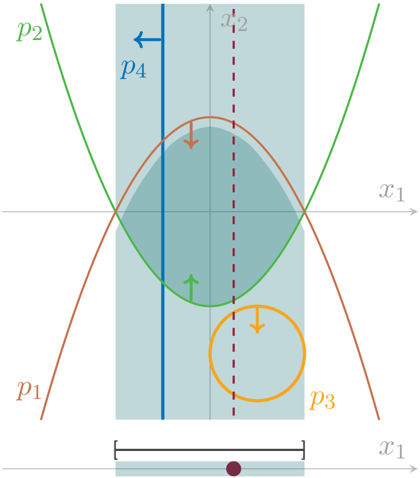
Sample $s = (x_1 : 0.25)$

Theorem

If $cl(S_1), \ldots, cl(S_k) \subseteq \mathbb{R}^j$ form a covering over S, then also over $cl(S)$.

Relaxation Non-trivial

$$\varphi \quad := \quad p_1 \leq 0 \quad \wedge \quad p_2 > 0 \quad \wedge \quad p_3 \geq 0 \quad \wedge \quad p_4 \geq 0$$



Relaxation Non-trivial

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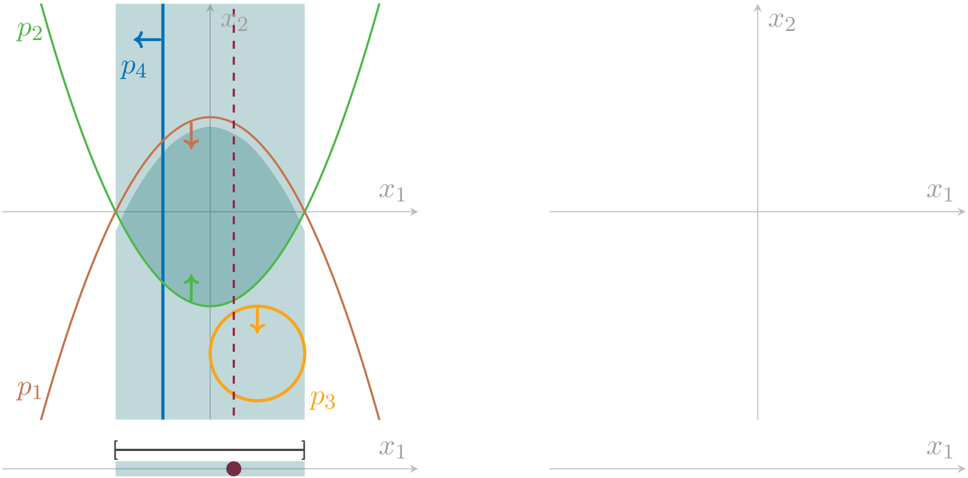

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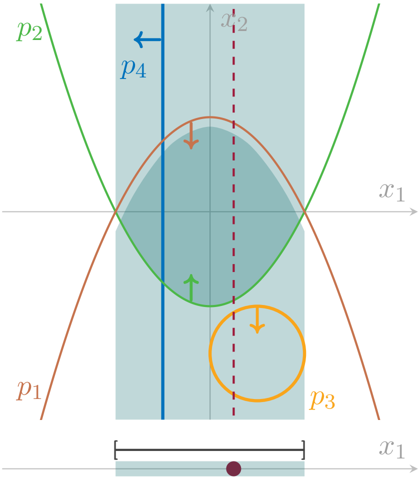

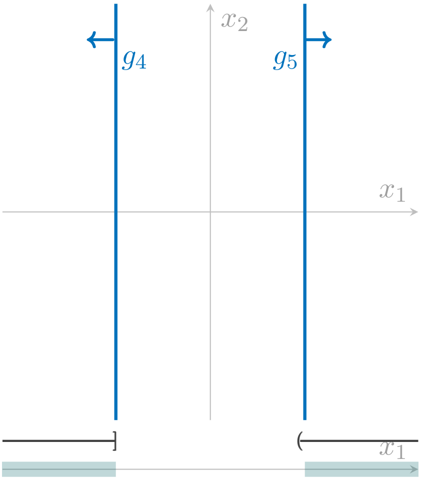

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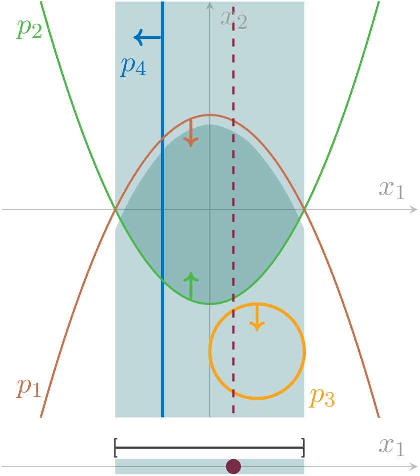

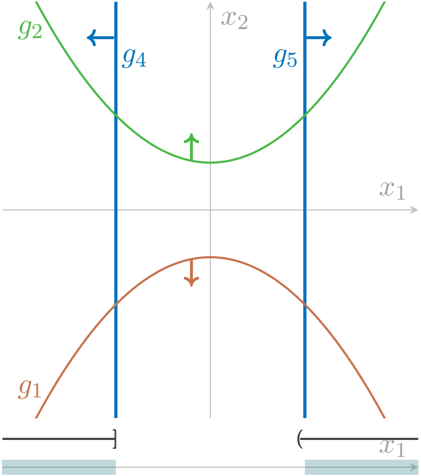

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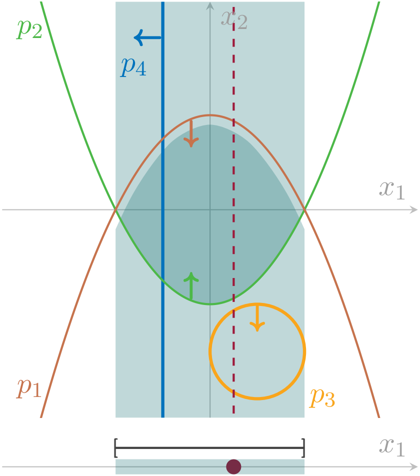

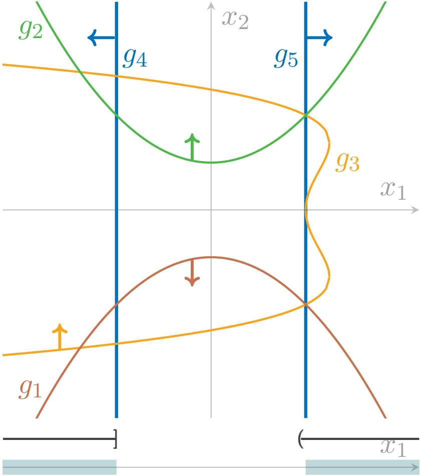

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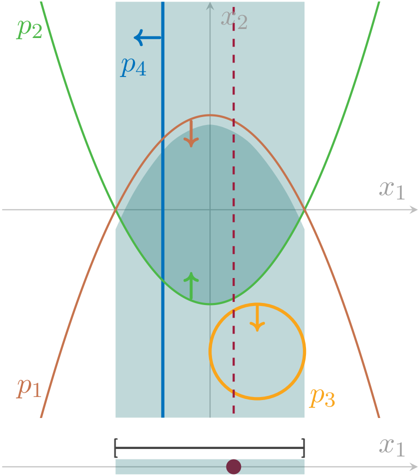

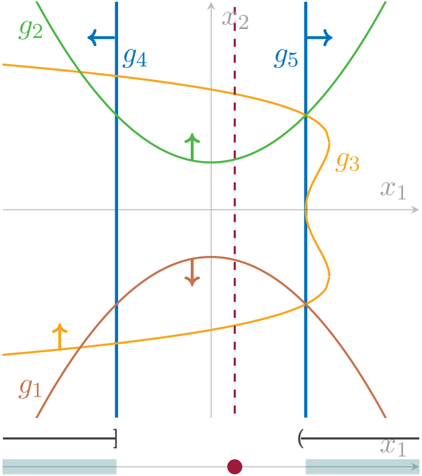

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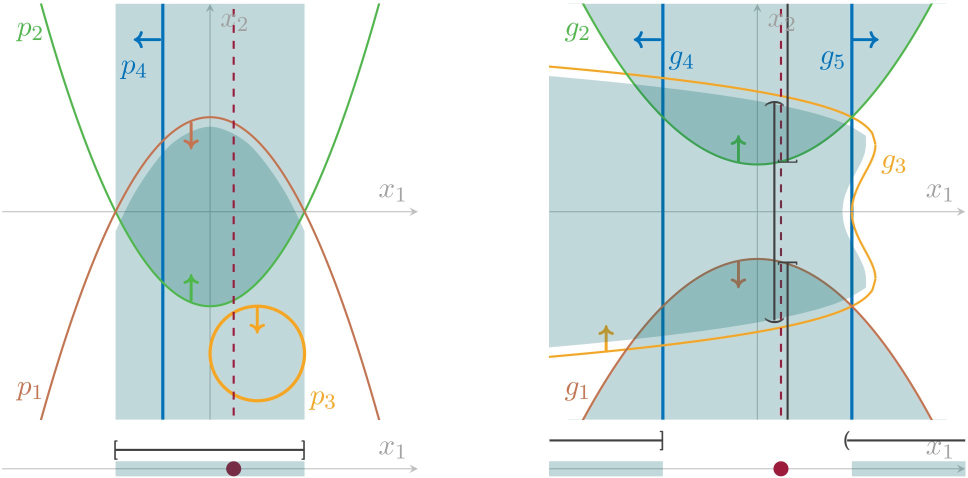

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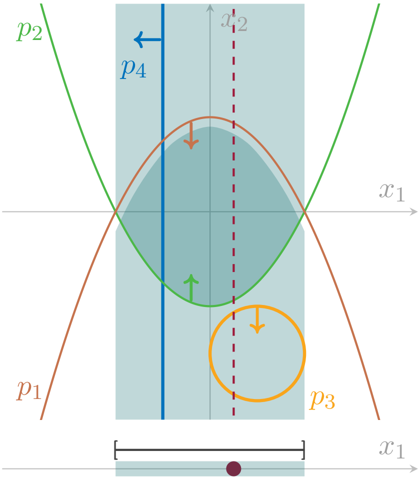

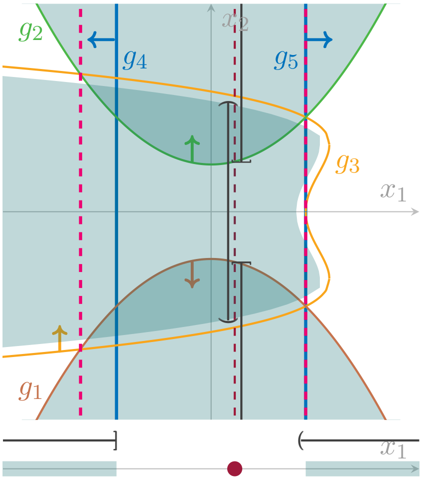

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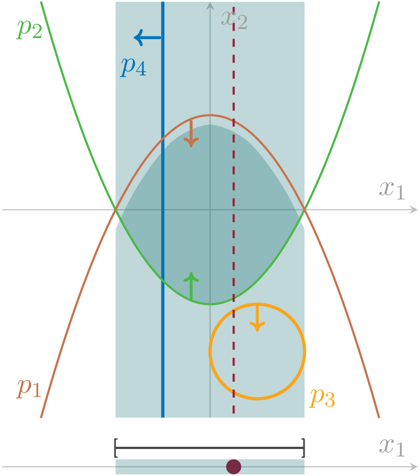

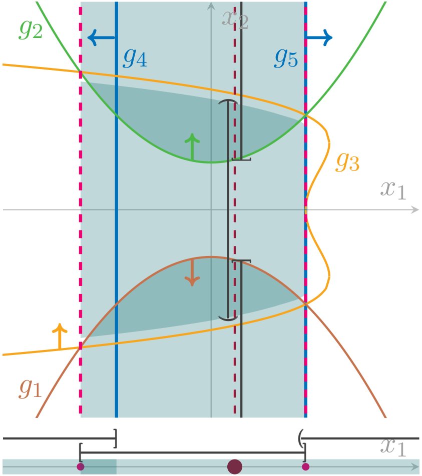

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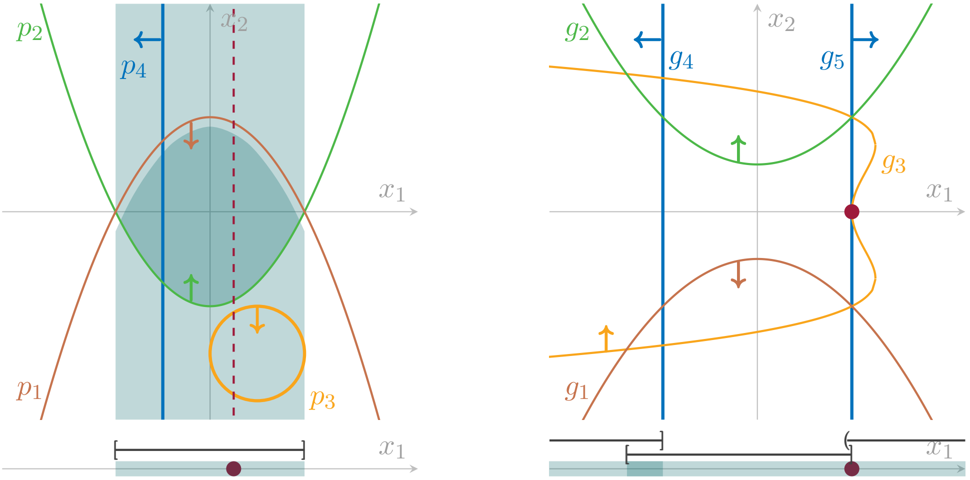

Algorithmic Adaption

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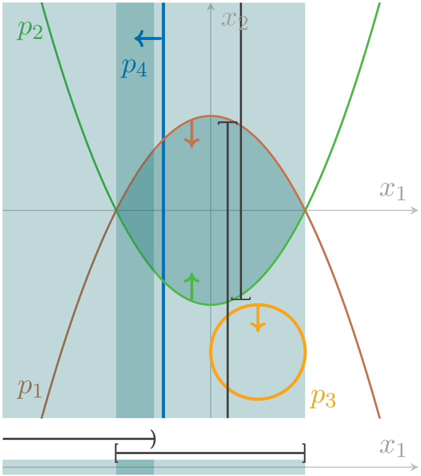

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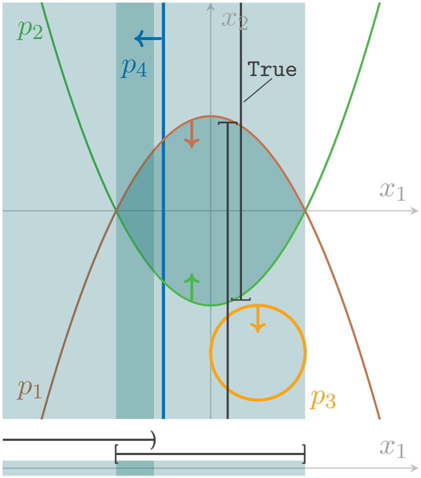

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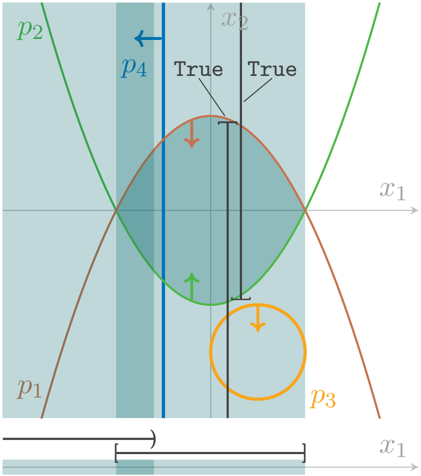

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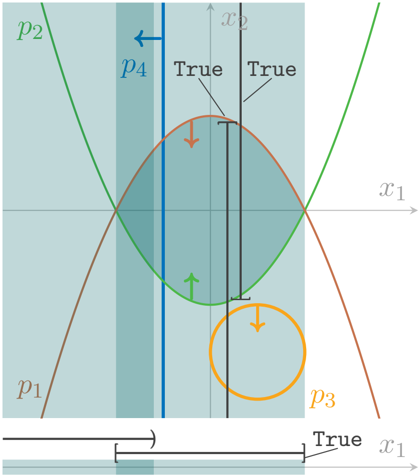

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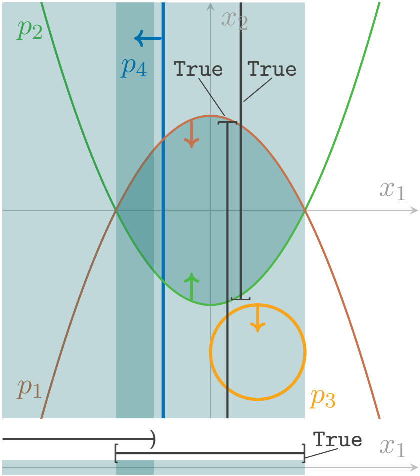

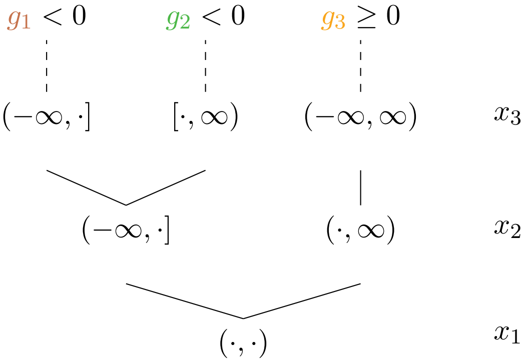

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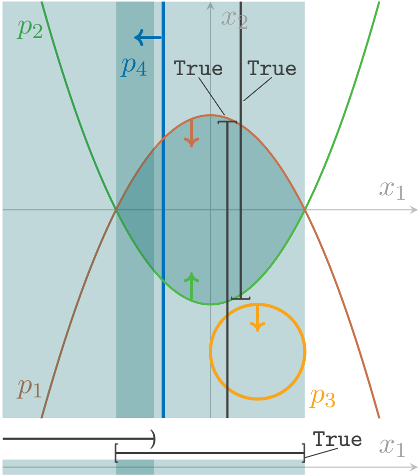

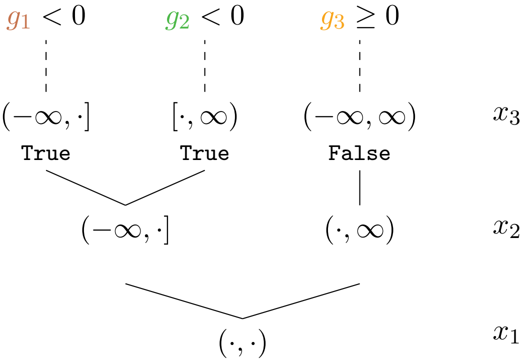

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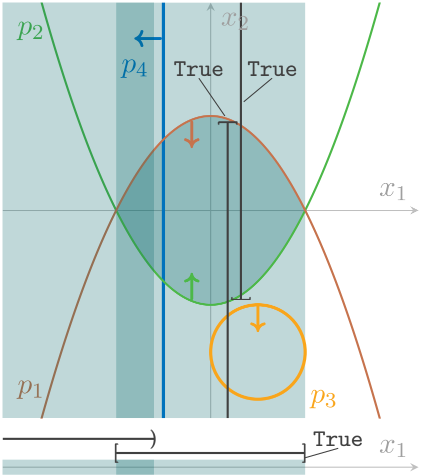

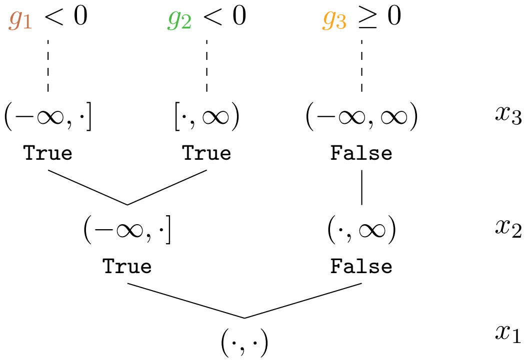

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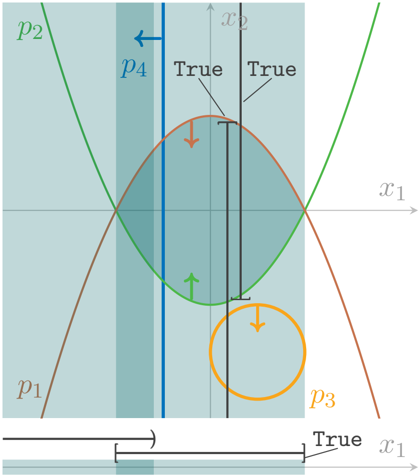

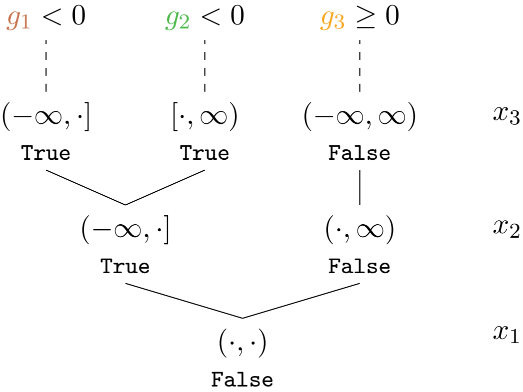

Experimental Results

Experimental Results

- `SMT-RAT`: CAIC in DPLL(T)

Experimental Results

- SMT-RAT: CAIC in DPLL(T)
- QF_NRA from SMT-LIB,
  11552 instances

Experimental Results

- SMT-RAT: CAIC in DPLL(T)
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Solver	SAT	UNSAT
Total	5069	5379
	(1104 unknown)	

Experimental Results

- SMT-RAT: CAIC in DPLL(T)
- QF_NRA from SMT-LIB, 11552 instances

Solver	SAT	UNSAT
CAlC	4553	4625
Total	5069	5379
	(1104 unknown)	

Experimental Results

- SMT-RAT: CAIC in DPLL(T)
- QF_NRA from SMT-LIB,
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Solver	SAT	UNSAT
CAIC	4553	4625
CAIC-I	4610	4648
Total	5069	5379
	(1104 unknown)	

+80

Experimental Results

- SMT-RAT: CAIC in DPLL(T)
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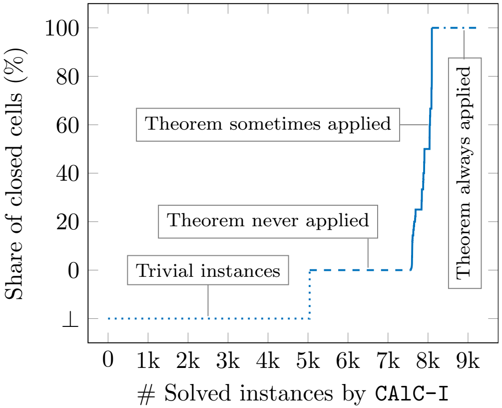

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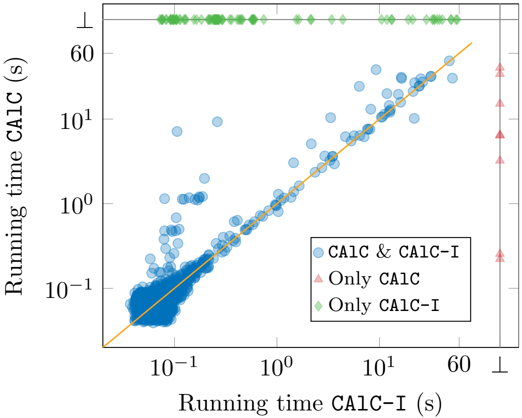
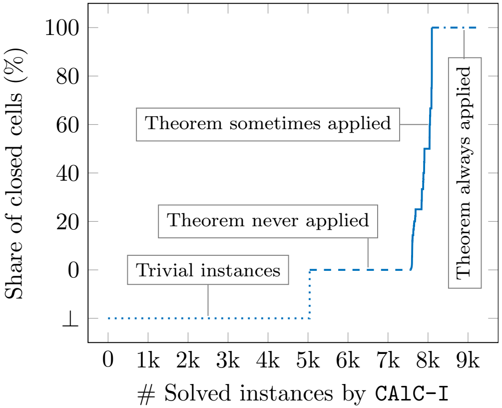
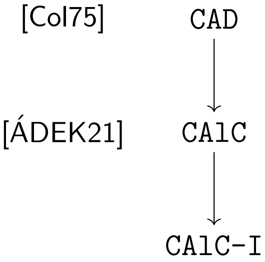




In a Nutshell



In a Nutshell

[Col75] CAD

↓

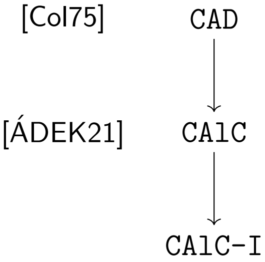
[ÁDEK21] CAlC

↓

CAlC-I

Future work

In a Nutshell



Future work

- Relaxation of the theorem conditions

In a Nutshell

[Col75] CAD

↓

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CAlC-I

Future work

- Relaxation of the theorem conditions
- Different covering heuristics

In a Nutshell

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CAlC-I

Future work

- Relaxation of the theorem conditions
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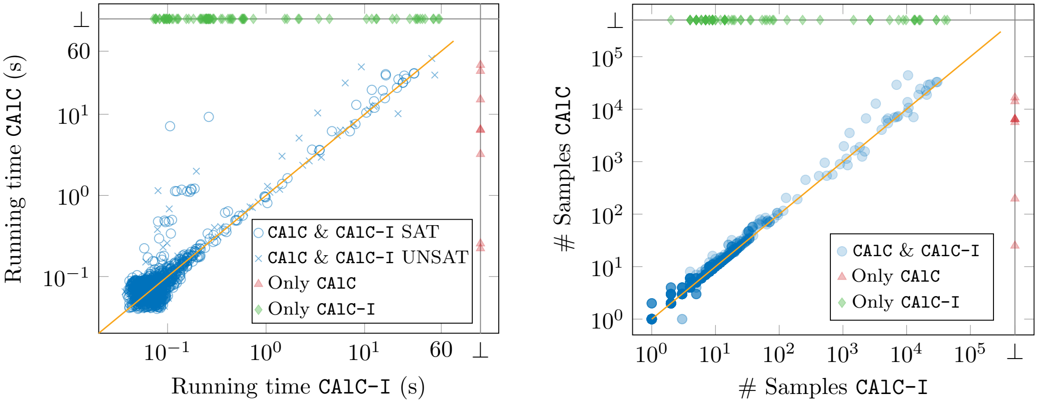
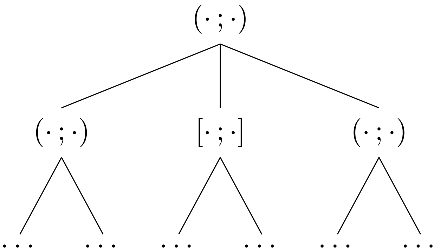
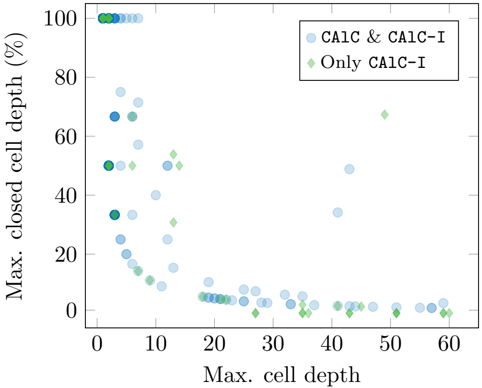
Experimental Results – Running Time & Samples



Experimental Results – Inheritance Depth





Experimental Results – Covering Heuristic

Flag focussed heuristic

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Flag focussed heuristic

- Modified interval selection

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- Prefer fully flagged coverings

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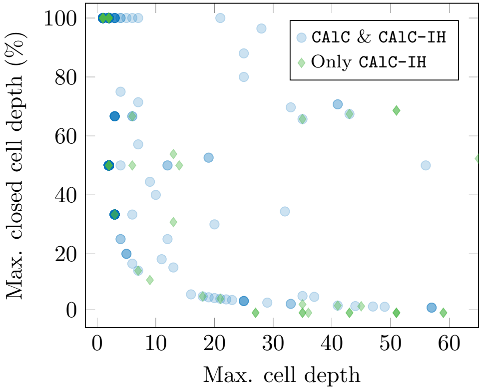

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Flag focussed heuristic

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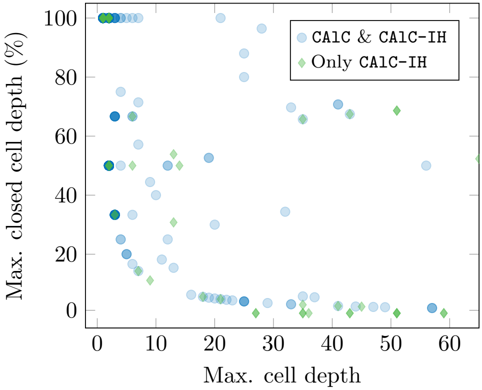
Total	5069	5379
	(1104 unknown)	



Experimental Results – Covering Heuristic

Flag focussed heuristic

- Modified interval selection
- Prefer fully flagged coverings

Solver	SAT	UNSAT
CAlC	4553	4625
CAlC-I	4610	4648
CAlC-IH	4609	4648
Total	5069	5379
	(1104 unknown)