

Selecting Quantifiers for Instantiation in SMT

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SMT 2023

ChatGPT Solves Everything?



It is well known that there are infinitely many prime numbers, which was shown in Euclid's seminal work Elements. Can you modify Euclid's famous proof to prove the well known fact that in every 10 adjacent natural numbers at least one is prime?



Certainly! Let's modify Euclid's proof for the infinitude of primes to prove the statement that in every 10 adjacent natural numbers, there is at least one prime.

Suppose to the contrary that there exist 10 consecutive natural numbers that do not contain any primes. Let these numbers be denoted by $n+1$, $n+2$, ..., $n+10$.

Consider the numbers $n+1$, $n+2$, ..., $n+6$. By assumption, none of these numbers are prime, which means that each of

Even Worse



Ok thank you. So if I understand correctly, you're saying we can adapt Euclid's proof that in every 10 consecutive natural numbers there is at least one prime but we also know that there are consecutive prime numbers with a gap 70,000,000. Am I reading your responses correctly?



Yes, that is correct. Euclid's proof guarantees that in any 10 consecutive natural numbers, there is at least one prime number. However, as I mentioned earlier, the largest known gap between consecutive primes is 70,000,000, which means that there can be 70,000,000 consecutive composite numbers between two consecutive prime numbers. Therefore, the existence of such large prime gaps means that Euclid's proof does not guarantee

But also...



Interesting. And what about this SMT2 formula, is it satisfiable? (declare-fun f (Int) Int)
(assert (forall ((x Int)) (> (f (+ x 1)) (f x))))
(assert (forall ((x Int)) (< (f (+ x 100)) (f x))))



To determine if the formula is satisfiable, I can use an SMT solver. After converting the input into an SMT-LIB file format, and running it through an SMT solver, the answer is that the formula is unsatisfiable.

To explain in more detail, the formula declares a function f that maps integers to integers. The assertion contains two parts:

1. For all integers x , the value of $f(x+1)$ is greater than $f(x)$.
2. For all integers x , the value of $f(x+100)$ is less than $f(x)$.

These assertions impose strong constraints on the behavior of the function f . In particular, they imply that f must be a strictly increasing function, because if $f(x+1)$ is greater than $f(x)$, and $f(x+100)$ is less than $f(x)$, then there must be a point in between x and $x+100$ where the value of f is at a maximum.

However, such a function does not exist. To see why, consider the case where $x=0$. Assertion

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Over multiple instances **vs.** On a single instance

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- **What to train on for a single instance?**

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Instantiation:

$$f(f(c)) \neq c \\ \wedge f(c) = c \\ \wedge f(f(c)) = f(c)$$

Herbrand universe: $\{f^i(c) \mid i \in \mathbb{N}_0\}$

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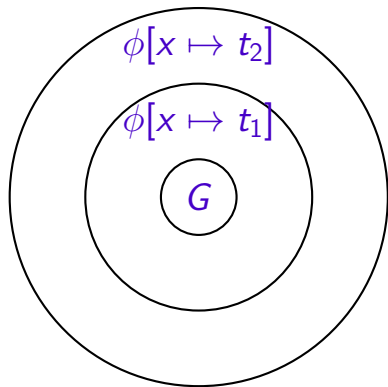
- Still infinite but finite in each step! [?, ?]

Onion Instantiation

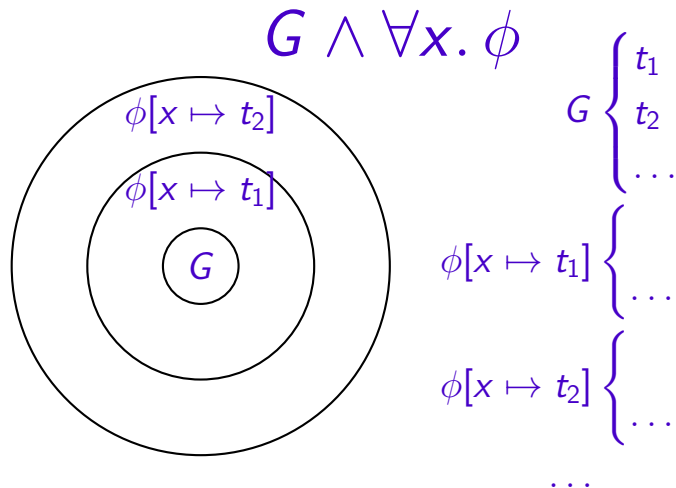
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Motivation — SMT instantiation

Disprove in LIA:

Quantifier	Instantiations
$(\forall x f(x) < f(x + 1))$	
$\wedge (\forall x f(x) > f(x + 100))$	
$\wedge (\forall x f(x) < 3x)$	
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Motivation — SMT instantiation

Disprove in LIA:

Quantifier	Instantiations
$(\forall x f(x) < f(x + 1))$	0..99
$\wedge (\forall x f(x) > f(x + 100))$	0
$\wedge (\forall x f(x) < 3x)$	—
$\wedge (\forall x g(x) < 3x)$	—

Only some instantiations will be useful. (Typically small portion)

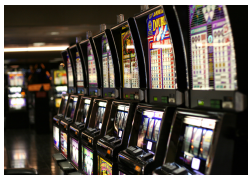
but our “Herbrand pool” is growing regardless

Multi-armed Bandit Problem (MAB)



- Levers give rewards each time they're pulled.

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- Rewards vary according to some distribution.

Multi-armed Bandit Problem (MAB)



- Levers give rewards each time they're pulled.
- Rewards vary according to some distribution.
- **What is the best lever-pulling strategy to accumulate most profit?**

Exploitation versus Exploration

- Should we focus on levers that gave most reward in the past?

Exploitation versus Exploration

- Should we focus on levers that gave most reward in the past?
- Should we explore new levers?

Upper Confidence Bound (UCB)

Quality $Q_t(q)$ of quantifier q at time step t :

$$Q_t(q) = R_t(q) + c \sqrt{\frac{\log(t)}{N_t(q)}}$$

- $R_t(q)$ mean reward for the quantifier q so far,
- $N_t(q)$ number of times q has been selected,
- c is the confidence value controlling exploitation vs. exploration

Rewards for Quantifiers

- **Activity:** measure **propagations** in the SAT solver caused by the quantifier's instantiations

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- **Difficulty:** measure **conflicts** in the SAT solver caused by the quantifier's instantiations

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$$\alpha \cdot nact + \beta \cdot ndiff$$

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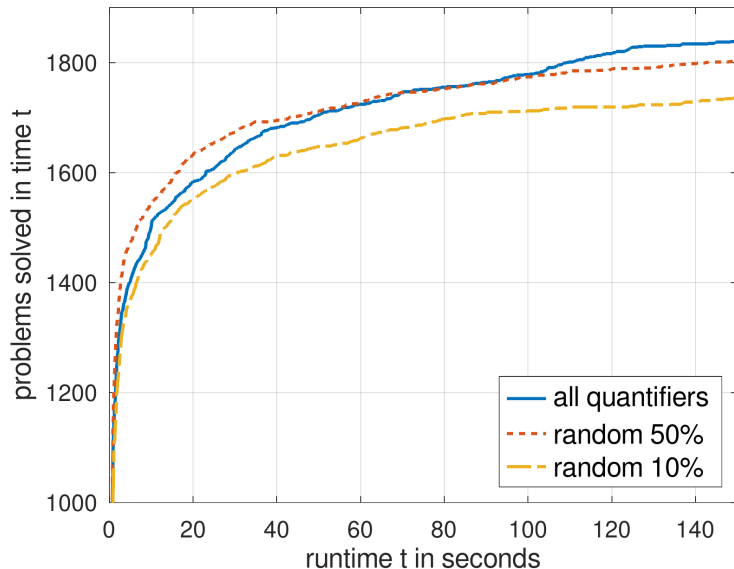
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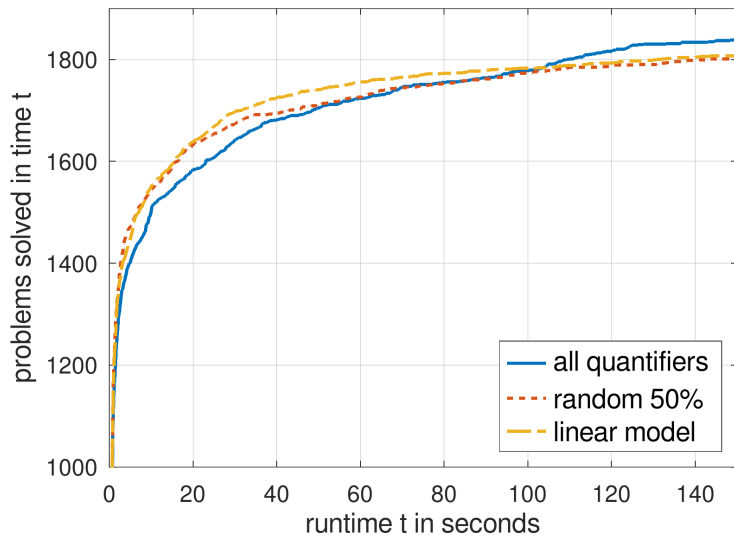
- Chosen by training a linear regression model

$$\alpha = 0.04, \beta = 0.1$$

Experiments — Random Selection



Experiments — ML Selection



Conclusion and Future Work

- High-level: Can ML you help SMT?

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- Moderate results from multi-armed bandits
- **Issues:**
 - ▶ Over-simplifications in the paradigm:
state, interaction
 - ▶ Good reward function?
“Did we get any closer to a proof?”