COUNTEREXAMPLE-GUIDED MODEL SYNTHESIS

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Introduction

- Counterexample-Guided
  Combine counterexample-guided quantifier instantiation with . . .
- Synthesis
  . . . syntax-guided synthesis to synthesize . . .
- Model
  . . . interpretations for Skolem functions.
- Quantified Bit-Vectors

Fixed-Size Bit-Vectors

Bit-Vector: vector of bits of a fixed size

- Constant values: $0011,\ 00000011,\ 3_{[8]}, \ldots$
- Variables: $x_{[16]},\ y_{[9]}, \ldots$
- Operators:
  - bitwise: $\sim,\ \&,\ |,\ \oplus,\ \ll,\ \gg, \ldots$
  - arithmetic: $+,\ -,\ *,\ /, \ldots$
  - predicates: $=,\ <,\ \leq, \ldots$
  - string operations: concat, extract, extension, ...

Example with Quantifiers $\quad \forall\, x_{[4]}\, \exists\, y_{[4]}\ .\ (x\ \&\ 1100) + y = 0000$

Quantified Bit-Vectors

State-of-the-Art

- Z3: Model-based quantifier instantiation (MBQI) [de Moura'09]
  - combined with E-matching
- CVC4: Counterexample-guided quantifier instantiation (CEGQI) [Reynolds'15]
- Q3B: BDD-based approach [Strejcek'16]
  - relies on simplifications, approximation techniques, variable ordering

Our approach

Counterexample-Guided Model Synthesis (CEGMS)

- ▷ Combines synthesis with variant of CEGQI

Counterexample-Guided Model Synthesis

Example $\varphi := \forall\, x_{[32]}\, \exists\, y_{[32]}\, .\, x + y = 0$

Skolem $\varphi_S := \forall\, x_{[32]}\, .\, x + f(x) = 0$

Ground Instances of φ_S

$\mathbf{x}$	$\mathbf{x} + \mathbf{f}(\mathbf{x}) = \mathbf{0}$
0	$0 + f(0) = 0$
1	$1 + f(1) = 0$
2	$2 + f(2) = 0$
$\vdots$	$\vdots$
2^{32}-1	$\ldots$

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2^{32}-1	$\ldots$

Function Table f

$\mathbf{x}$	$\mathbf{f(x)}$
0	0
1	-1
2	-2
$\vdots$	$\vdots$
2^{32}-1	-(2^{32}-1)

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Goal

$f := \lambda x.-x$

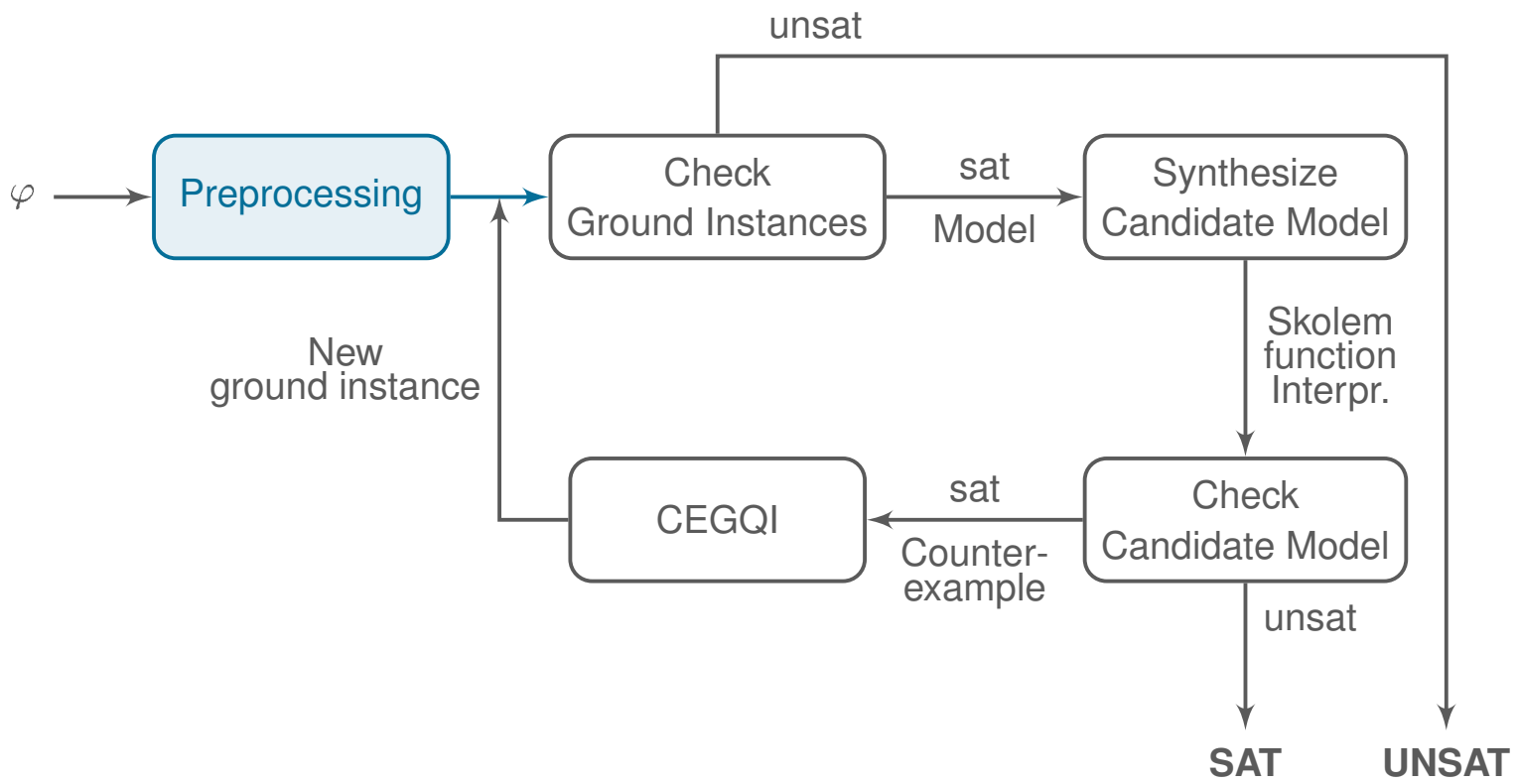
$\downarrow$

$\forall x_{[32]}\, .\, x + -x = 0$ ✓

How?

Synthesize + Refine

Workflow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Candidate Model
Skolem
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Interpr.
New
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CEGQI
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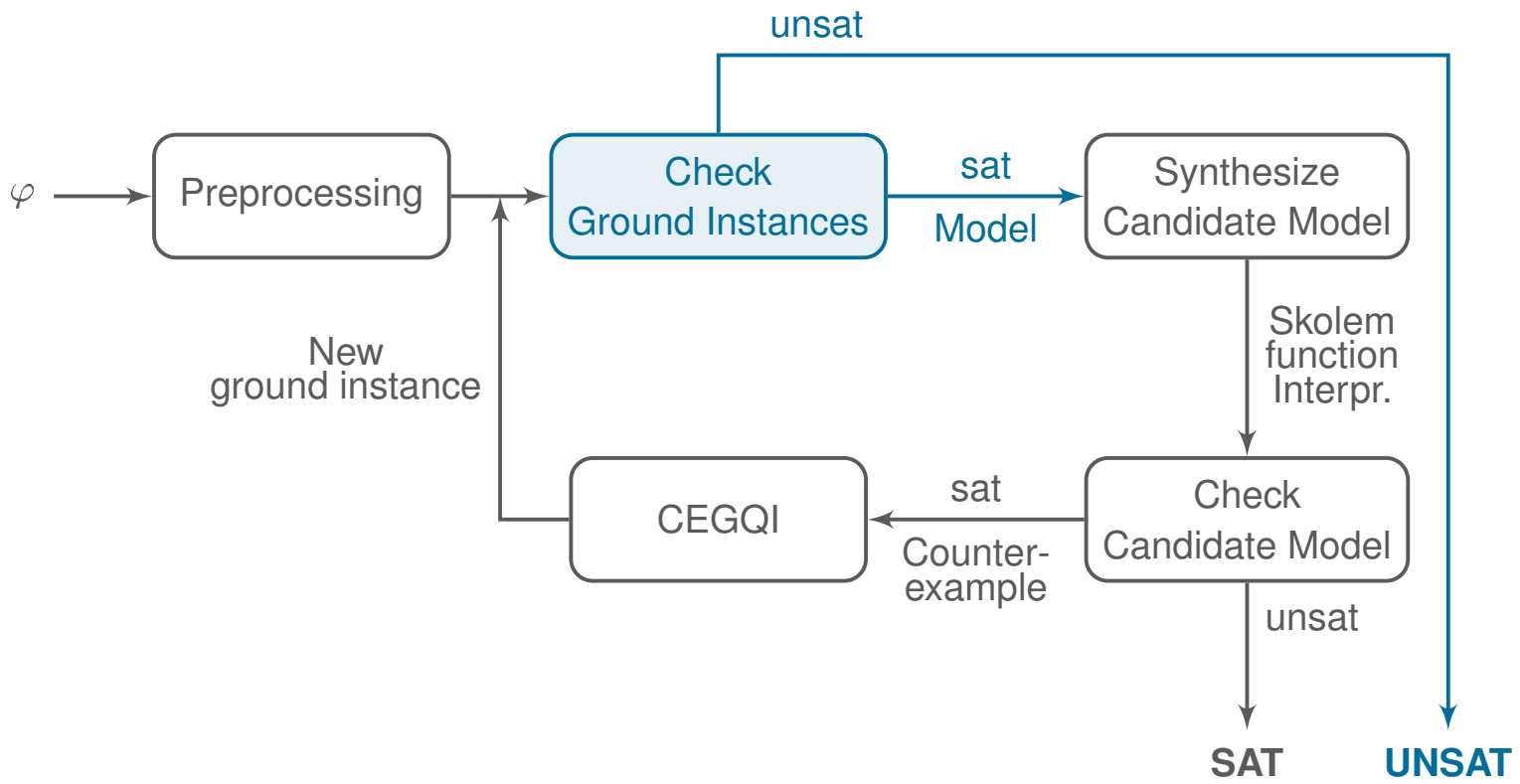
Workflow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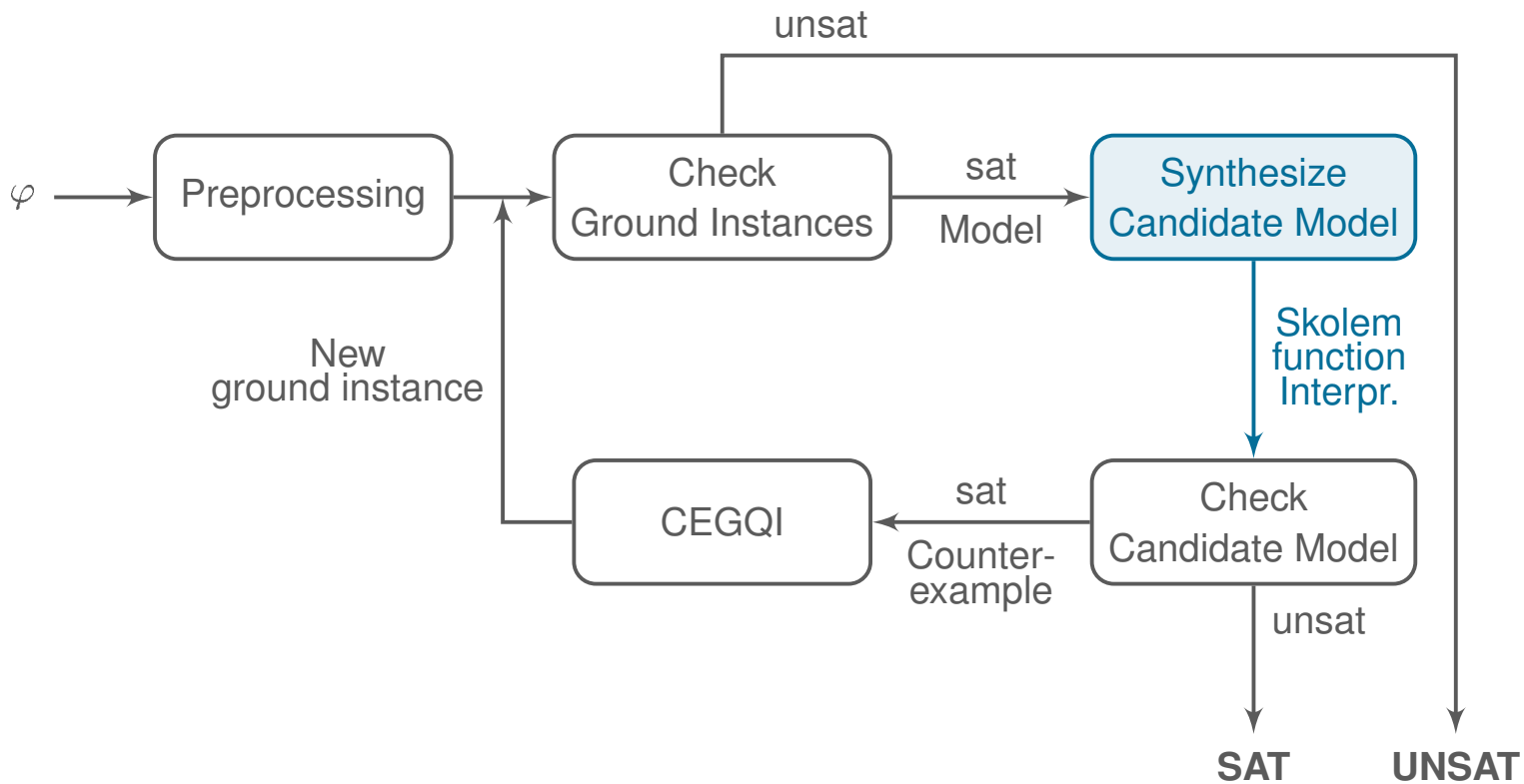
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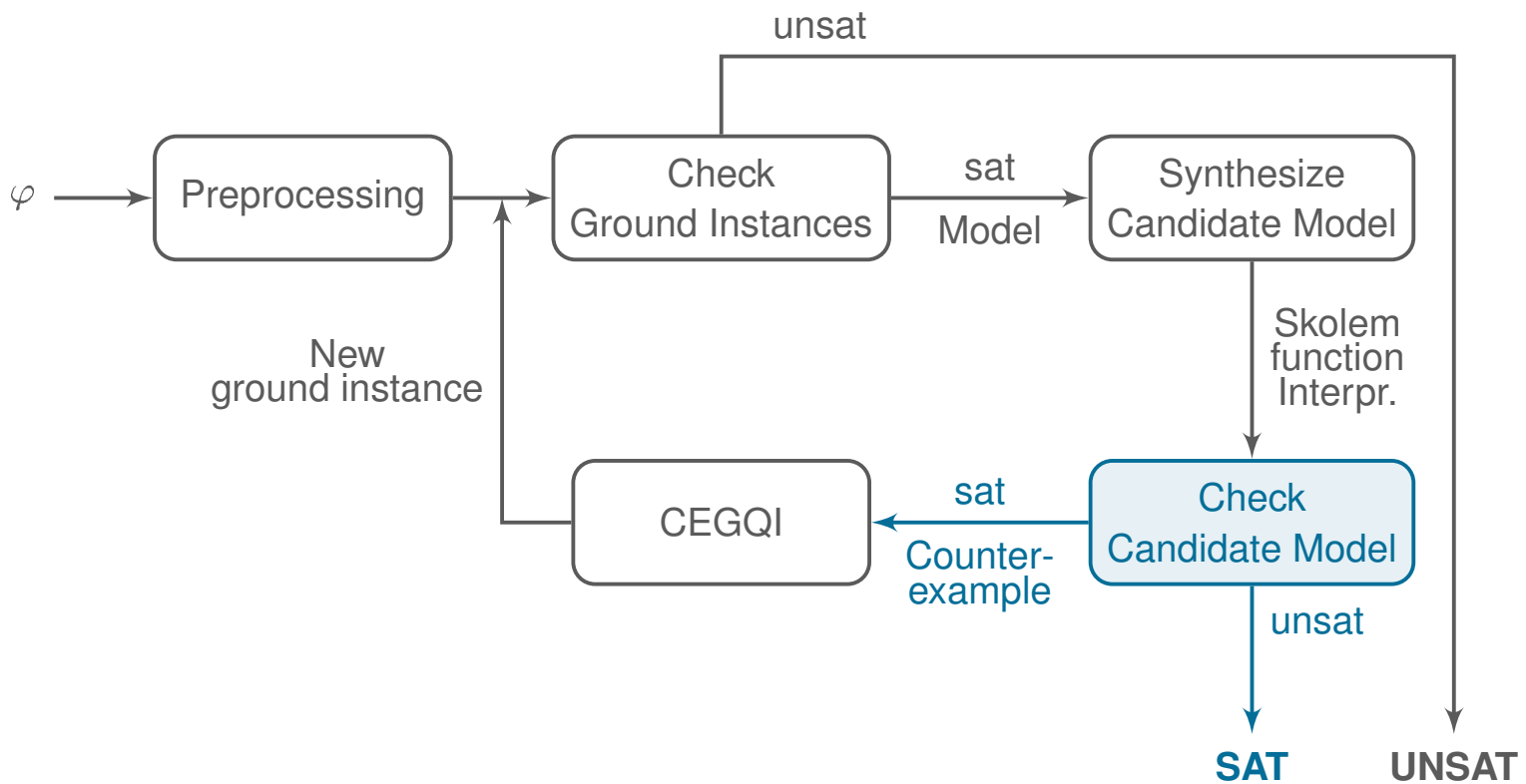
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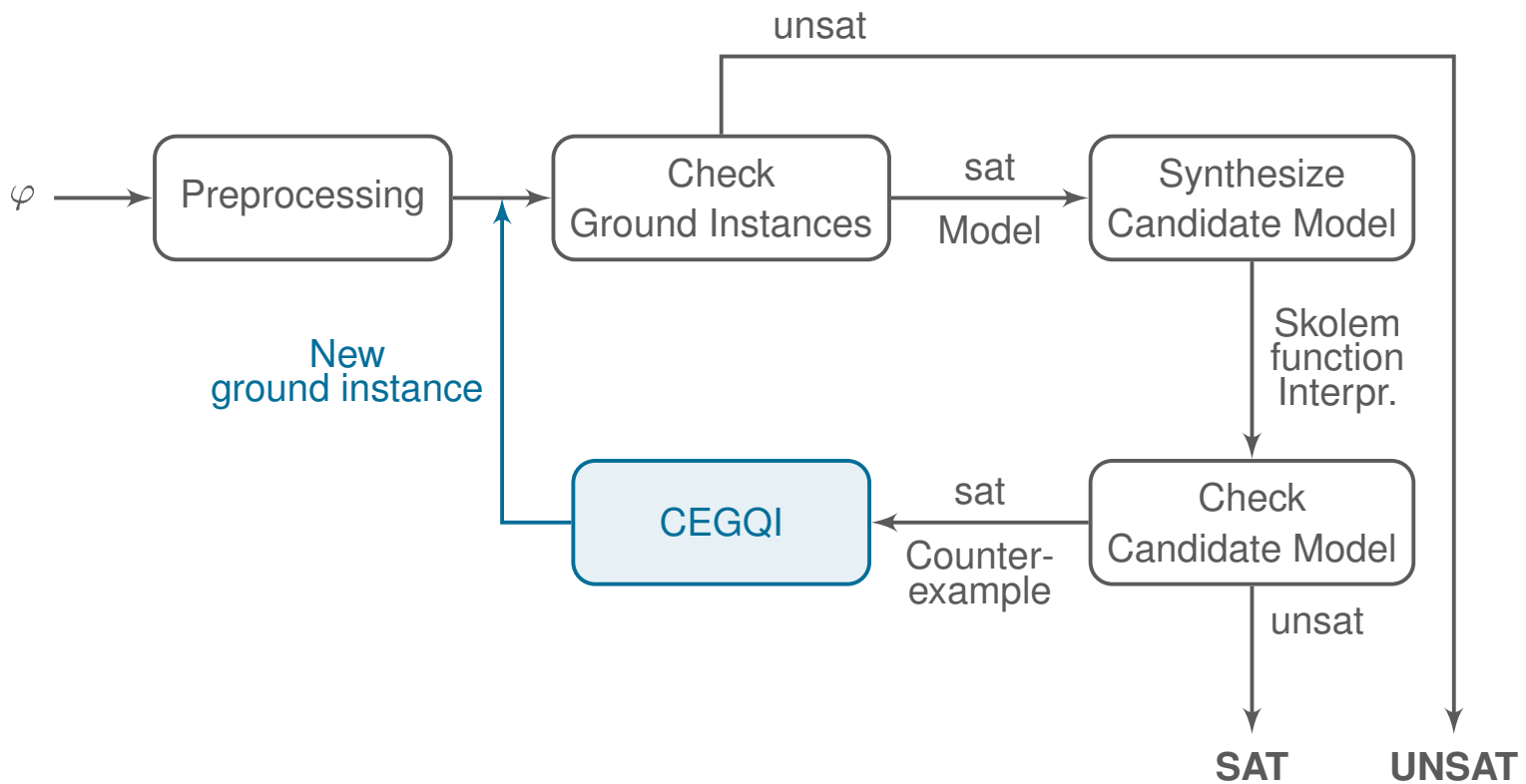
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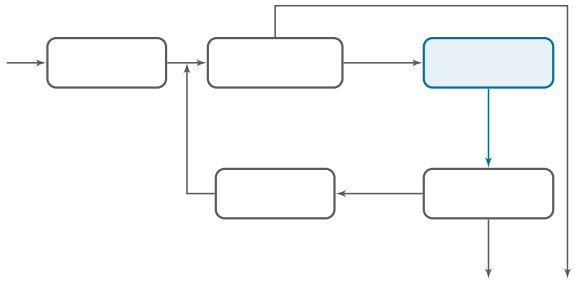
Synthesis of Candidate Models



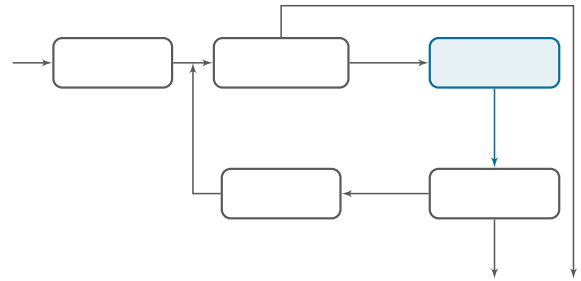
Enumerative Learning [Alur'13]

- enumerate expressions based on a syntax/grammar
  - ▷ start with smallest expressions (inputs)
  - ▷ enumerate expressions in increasing size
  - ▷ prune enumeration space
- check if expression conforms to some set of test cases
- return candidate expression if set of test cases is covered

Synthesis of Skolem Functions

- apply enumerative learning to each Skolem symbol
- use set of ground instances G as "test cases"
- substitute Skolem symbol with candidate expression in G
- evaluate $g_i \in G$
  - ▷ return candidate interpretation if all $g_i \in G$ are satisfied

Synthesis of Candidate Models (cont.)



Example

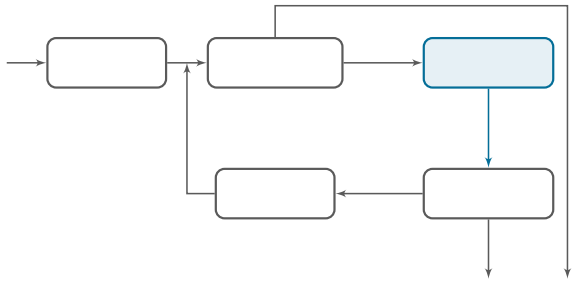
Inputs: x, y, 0, 1 Operators: $=$, $+$, $\&$, ite

Size	Enumerated Expressions
1	x y 0 1
2	$x = y$ $x = 0$ $x = 1$ $y = x$ $y = 0$ $y = 1$ $x + y$ $\cdots$
3	$(x + y) + x$ $(x + y) + y$ $(x + y) + 1$ $\cdots$
4	$(x = y)\&(x = 0)$ $(x = y)\&(x = 1)$ $\cdots$ $\text{ite}(x = y, x, y)$ $\cdots$
$\vdots$	

Expression Size: $\text{size}(x = y) := \text{size}(x) + \text{size}(y)$

▷ Large enumeration space!

Pruning Enumeration Space



Idea: discard “similar” expressions

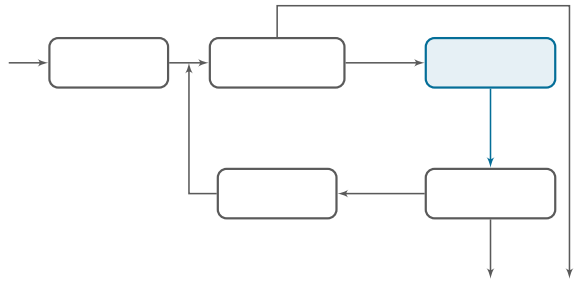
While enumerating expressions . . .

1. generate signature for each expression
2. if signature already cached discard expression
3. else cache signature

Signature Computation of Expression e

- set of ground instances $G := \{g_1, \ldots, g_n\}$
- substitute Skolem symbol f in G with e
- evaluate resulting ${g_1}', \ldots, {g_n}'$
- store evaluations (Boolean values) as vector of size n (= signature)
  - ▷ if every value is true, e is a candidate interpretation for f

Example: Synthesis



Example: $z = \min(x, y)$

$$\varphi := \forall x\, y \exists z\, .\, (x < y \rightarrow z = x) \wedge (x \geq y \rightarrow z = y)$$

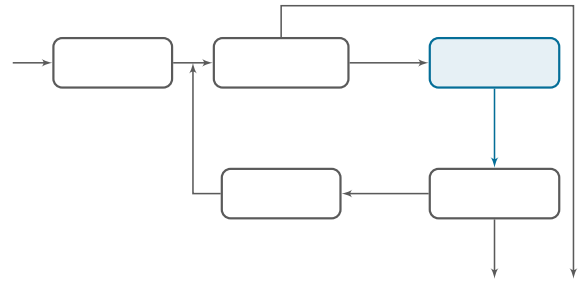
$$\varphi_S := \forall x\, y\, .\, (x < y \rightarrow f_z(x, y) = x) \wedge (x \geq y \rightarrow f_z(x, y) = y)$$

Inputs for f_z $\{ x,\ y \}$

Operators $\{ =,\ <,\ \geq,\ \wedge,\ \rightarrow,\ ite \}$

Ground Inst. G $\{ f_z(0, 0) = 0,$
$f_z(0, 1) = 0,$
$f_z(2, 1) = 1 \}$

Example: Synthesis cont.

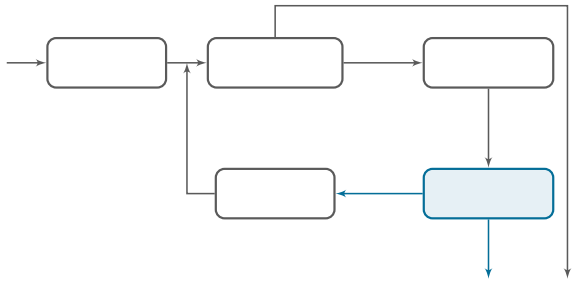


Size	Enumerated Expressions
1	$x,\ y$
2	$x = y,\ x < y,\ y < x,\ x \geq y,\ y \geq x$
3	–
4	$(x = y \wedge x < y),\ \ldots,\ (x = y \rightarrow x < y),\ \ldots,\ ite(x < y, x, y)$

Signature of Candidate $ite(x < y, x, y)$

$$\underbrace{ite(0 < 0, 0, 0) = 0}_{\top},\ \underbrace{ite(0 < 1, 0, 1) = 0}_{\top},\ \underbrace{ite(2 < 1, 2, 1) = 1}_{\top}$$

Example: Check Candidate Model



Candidate Model $\{f_z := \lambda\, x\, y\, .\, ite(x < y, x, y)\}$

Check

$$\neg\varphi_S[\lambda\, x\, y\, .\, ite(x < y, x, y)/f_z]$$

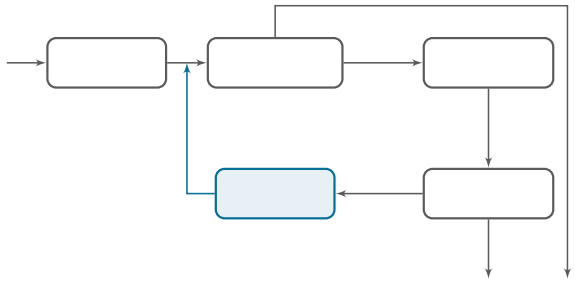
$$\equiv \exists\, x\, y\, .\, (x < y \wedge ite(x < y, x, y) \neq x) \vee (x \geq y \wedge ite(x < y, x, y) \neq y)$$

SMT Solver Check

$$\underbrace{(a < b \wedge ite(a < b, a, b) \neq a)}_{\bot} \vee \underbrace{(a \geq b \wedge ite(a < b, a, b) \neq b)}_{\bot}$$

- unsat: candidate model is valid
- sat: found counterexample, refine

Example: Refinement



Assume Candidate Model

$$\{ f_z := \lambda\, x\, y\, .\, x \}$$

SMT Solver Check

$$\underbrace{(a < b \wedge a \neq a)}_{\bot} \vee \underbrace{(a \geq b \wedge a \neq b)}_{\top}$$

- Solver returns sat, candidate model is invalid
- Solver produces counterexample $\{\, a = 1, b = 0 \,\}$

Add New Instance of φ_S to G

$$G := G \cup \{\varphi_S[1/x, 0/y]\}$$

Dual Counterexample-Guided Model Synthesis

Idea Find instantiation for ∀-variables s.t. formula is unsatisfiable.

How Apply CEGMS to the dual formula $\neg\varphi$

Duality CEGMS($\neg\varphi$) sat $\rightsquigarrow$ CEGMS(φ) unsat

CEGMS($\neg\varphi$) unsat $\rightsquigarrow$ CEGMS(φ) sat

Original $$\varphi \coloneqq \exists\, a\, b\, c \,\forall\, x \;.\; \underbrace{(a * c) + (b * c) \neq (x * c)}_{\text{unsat with } \varphi[a+b/x]}$$

Dual $$\neg\varphi \coloneqq \forall\, a\, b\, c \,\exists\, x \;.\; \underbrace{(a * c) + (b * c) = (x * c)}_{\text{sat with } \neg\varphi[a+b/x]}$$

- CEGMS(φ) and CEGMS($\neg\varphi$) can be executed in parallel

Experiments

	SMT-LIB (191)				**New**[1] (4838)			
	Solved	Sat	Unsat	Time [s]	Solved	Sat	Unsat	Time [s]
Boolector	142	51	91	59529	4527	465	4062	389020
Boolector+s	164	72	92	32996	4526	467	4059	390613
Boolector+d	162	67	95	35877	4572	**518**	4054	342412
Boolector+ds	**172**	**77**	**95**	**24163**	**4704**	517	**4187**	**187411**

Boolector . . . CEGQI only +**s** . . . synthesis +**d** . . . dual (parallel)

Limits 1200 seconds CPU time, 7GB memory

[1] LIA, LRA, NIA, NRA SMT-LIB benchmarks translated to BV

Experiments

	SMT-LIB (191)				**New** (4838)			
	Solved	Sat	Unsat	Time [s]	Solved	Sat	Unsat	Time [s]
Boolector+ds	172	77	**95**	24163	4704	**517**	4187	187411
CVC4	145	64	81	57652	4362	339	4023	580402
Q3B	**187**	**93**	94	**9086**	4367	327	4040	581252
Z3	161	69	92	37534	4732	476	**4256**	**133241**

Limits 1200 seconds CPU time, 7GB memory

Experiments

Synthesis Overhead (Runtime)

- up to 75% on solved benchmarks
- up to 98% on unsolved benchmarks

Refinement Iterations

- up to 300 iterations on solved benchmarks
- up to 9400 iterations on unsolved benchmarks

Synthesized Terms

- c
- x_i
- $(x_i \ op \ x_j)$
- $(c \ op \ x_i)$
- $\sim(c * x_i))$
- $(x_i + (c + \sim x_j))$

x_i . . . universal variables, c . . . constant value, op . . . bit-vector operator

Conclusion

- simple approach for solving quantified bit-vectors
  - only requires two instances of ground theory solvers
  - enumerative learning algorithm straightforward to implement
- competitive with the state-of-the-art in solving BV
  - no simplification techniques yet
  - no E-matching or other quantifier instantiation heuristics
- future directions
  - improve synthesis approach
    - employ divide and conquer approach from [Alur'17]
    - employ other synthesis approaches?
  - generalize counterexamples via synthesis
  - model reconstruction from unsatisfiable dual formulas
  - useful for other theories?

References I

[Alur'17] Rajeev Alur and Arjun Radhakrishna and Abhishek Udupa. Scaling Enumerative Program Synthesis via Divide and Conquer. TACAS, Pages 319-336. 2017

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[Reynolds'15] Andrew Reynolds and Morgan Deters and Viktor Kuncak and Cesare Tinelli and Clark W. Barrett. Counterexample-Guided Quantifier Instantiation for Synthesis in SMT. CAV, Pages 198-216. 2015

References II