# Stochastic Local Search for SMT: a Preliminary Report (Extended Abstract)

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# Outline

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# Motivations and Goals

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- "Lazy" approach to SMT: integration of DPLL and a $\mathcal{T}$-solver.
- SAT: stochastic local-search (SLS) procedures sometimes outperform DPLL on satisfiable instances (unstructured problems).
- Therefore, it is a natural research question to wonder whether SLS can be exploited successfully also inside SMT tools.

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# Stochastic Local Search for SAT

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- typically incomplete
- start at some location of the search space
- iteratively move from the current to a neighbouring location taking decisions on the base of local information

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- successfully applied to $\mathcal{NP}$-complete decision problems
- for SAT, WalkSAT is a popular (family of) SLS-based algorithm(s)

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# The WalkSAT Family Schema

**Require:** CNF formula $\varphi$, MAX_TRIES, MAX_FLIPS
1: **for** $i = 1$ to MAX_TRIES **do**
2: $\mu \leftarrow$ INITIALTRUTHASSIGNMENT($\varphi$)
3: **for** $j = 1$ to MAX_FLIPS **do**
4: **if** ($\mu \models \varphi$) **then**
5: **return** SAT
6: **else**
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8: $\mu \leftarrow$ NEXTTRUTHASSIGNMENT($\varphi, c, \mu$)
9: **end if**
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# SMT from a SAT perspective

SMT on $\varphi$: solve a *partially-invisible* SAT formula $\varphi^p \wedge \tau^p$ s.t.

- $\varphi^p$ is the "visible" part (Boolean abstraction of $\varphi$)
- $\tau^p$ is the "invisible" part (the B.a. of the set of $\mathcal{T}$-lemmas induced by the theory $\mathcal{T}$ on the atoms of $\varphi$)

In traditional "lazy" SMT solvers

- DPLL solver knows $\varphi^p$ but not $\tau^p$ ($\mathcal{T}$-solver "knows" $\tau^p$)
- whenever $\mu^p \models \varphi^p$, $\mathcal{T}$-solver checks whether $\mu^p$ falsifies $\tau^p$ and returns one falsified clause $c^p$ in $\tau^p$
- DPLL uses $c^p$ to drive the future search (optionally add it to $\varphi^p$)

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**Require:** SMT($\mathcal{T}$) CNF formula $\varphi$, MAX_TRIES, MAX_FLIPS

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Intuition: $\mathcal{T}$-solver plays the role of CHOOSEUNSATISFIEDCLAUSE on $\varphi^p \wedge \tau^p$ when no unsatisfied clause is found in $\varphi^p$.

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# Preprocessing

Simplify the input formula $\varphi$ through:

## unit propagation

- unit-propagate each literal occurring as unit clause in $\varphi$
- add to $\varphi$ the conjunction of all non-propositional unit literals eliminated

$\Longrightarrow$ eliminates purely-propositional variables (and possibly others)

## static learning

- augment $\varphi$ with short "obvious" $\mathcal{T}$-lemmas generated without invoking the $\mathcal{T}$-solver (e.g., $\neg(x > y) \vee \neg(y > z) \vee (x > z)$)

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# Learning and Unit simplification

## Learning

- *Learn* the $\mathcal{T}$-lemmas generated by the $\mathcal{T}$-solver

$\Longrightarrow$ avoid finding the same $\mathcal{T}$-conflict multiple times

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# Filtering the assignments given to $\mathcal{T}$-solver

**pure-literal filtering**

If non-Boolean $\mathcal{T}$-atoms occur only positively [negatively] in the original formula $\varphi$, drop every negative [positive] occurrence of them from $\mu^p$

$\Longrightarrow$

- reduce the work of $\mathcal{T}$-solver by removing $\mathcal{T}$-atoms from the assignment $\mu^p$ to be checked,
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Idea: Since $\mu^p$ are total assignments, they may be $\mathcal{T}$-inconsistent for several different reasons.
$\Longrightarrow$ Learn more than one $\mathcal{T}$-lemma.

## Multiple learning

- invoke the $\mathcal{T}$-solver on $\mu$ to find a new conflict set $\eta$
- if a conflict set $\eta$ is found,
  - unit-simplify and learn the $\mathcal{T}$-lemma $\neg\eta$
  - compute a sub-assignment $\mu'$ by removing from $\mu$ (part or all) the literals occurring in $\neg\eta$
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Note: ChooseUnsatisfiedClause picks randomly one of the $\neg\eta$s

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# Efficient $\mathcal{T}$-solvers for LS

## DPLL-based SMT solvers

- truth assignments are updated in a stack-based manner
- $\mathcal{T}$-solvers typically *incremental* and *backtrackable*

## SLS-based SMT solvers

- truth assignments are updated by flipping an arbitrary literal
- *backtrackable* $\mathcal{T}$-solvers are of little use
- it is desirable to be able to remove and add arbitrary literals from a $\mathcal{T}$-solver without the need of resetting its internal state (some $\mathcal{T}$-solvers for $\mathcal{DL}$ and $\mathcal{LA}(\mathbb{Q})$ have this capability)

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# A prototype SLS-based SMT solver for $\mathcal{LA}(\mathbb{Q})$: WalkSMT

## Implementation

- implemented in C++
- built on top of UBCSAT SLS platform [Tompkins & Hoos'04]
  - implements many existing SLS procedures
  - after various attempts and preliminary testing, we selected the AdaptiveNovelty+ procedure
- uses MathSAT preprocessor and $\mathcal{LA}(\mathbb{Q})$-solver
- implements the optimizations previously described

## Execution

- on a 2.66GHz 4GB RAM Xeon machine on linux
- 600s timeout
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# Comparison: WALKSMT vs. MathSAT

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- LEARNING-WalkSMT combines BASIC-WALKSMT with preprocessing, unit simplification and learning
- BEST-WalkSMT extends LEARNING-WalkSMT with multiple learning, pure-literal filtering optimizations

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# SMT-LIB Instances

"industrial" formulas on $\mathcal{LA}(\mathbb{Q})$ (encoding real-word problems)

- 7 categories of formulas
- 20 formulas per category

**Plots**

- scatter-plots with logscale
- for WALKSMT, 3 runs with different seeds for each formula

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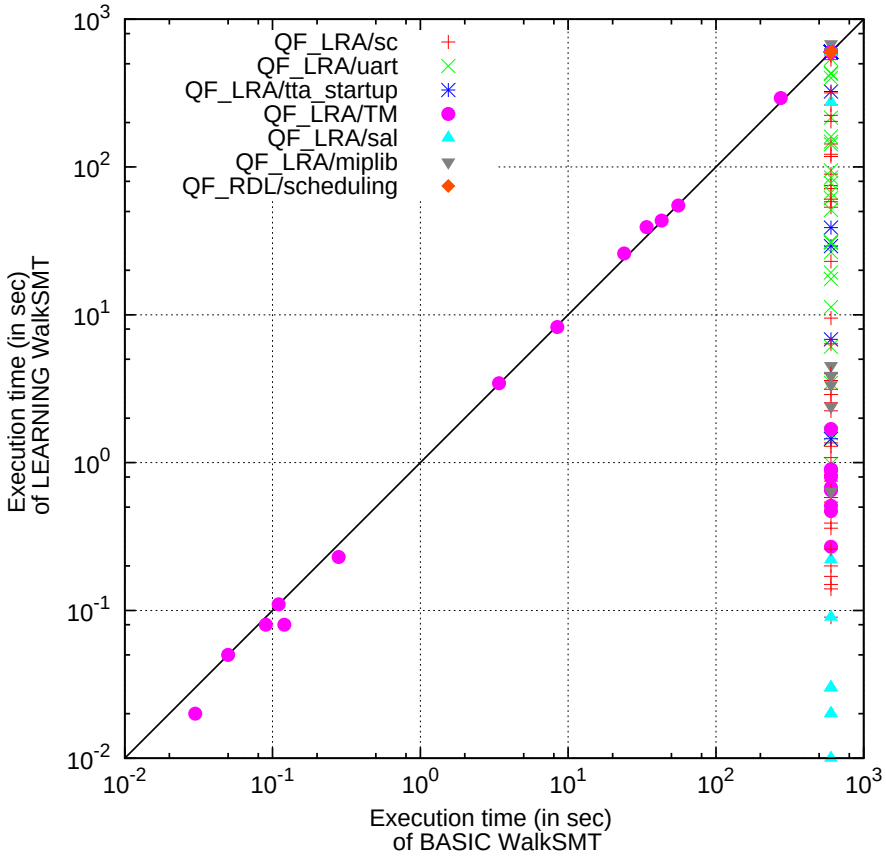
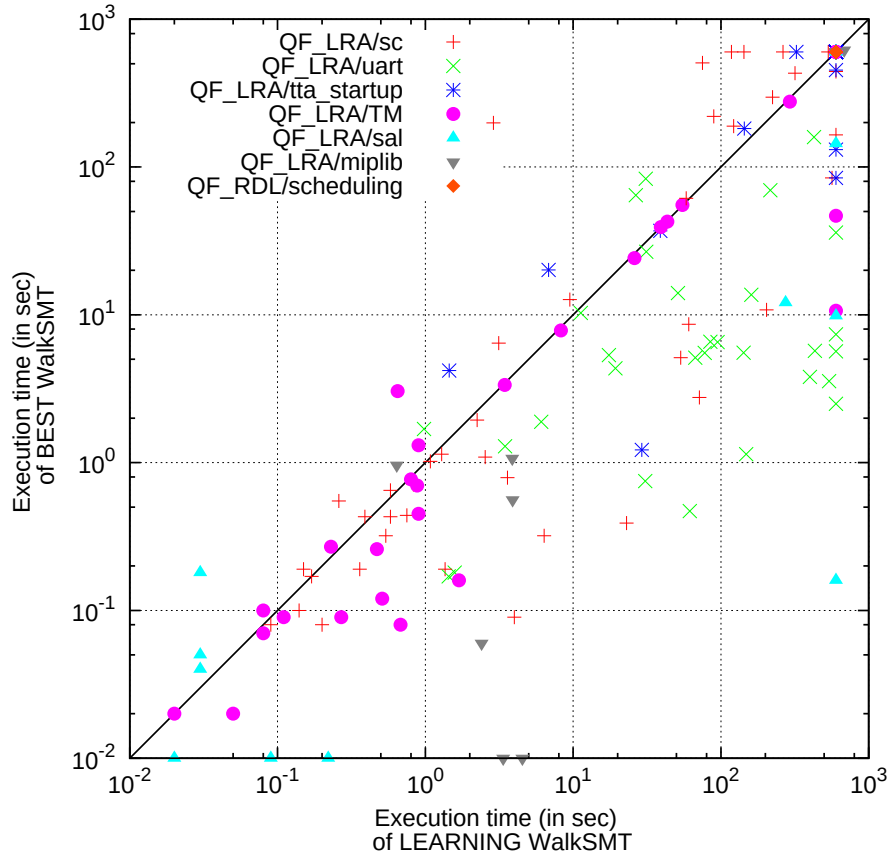
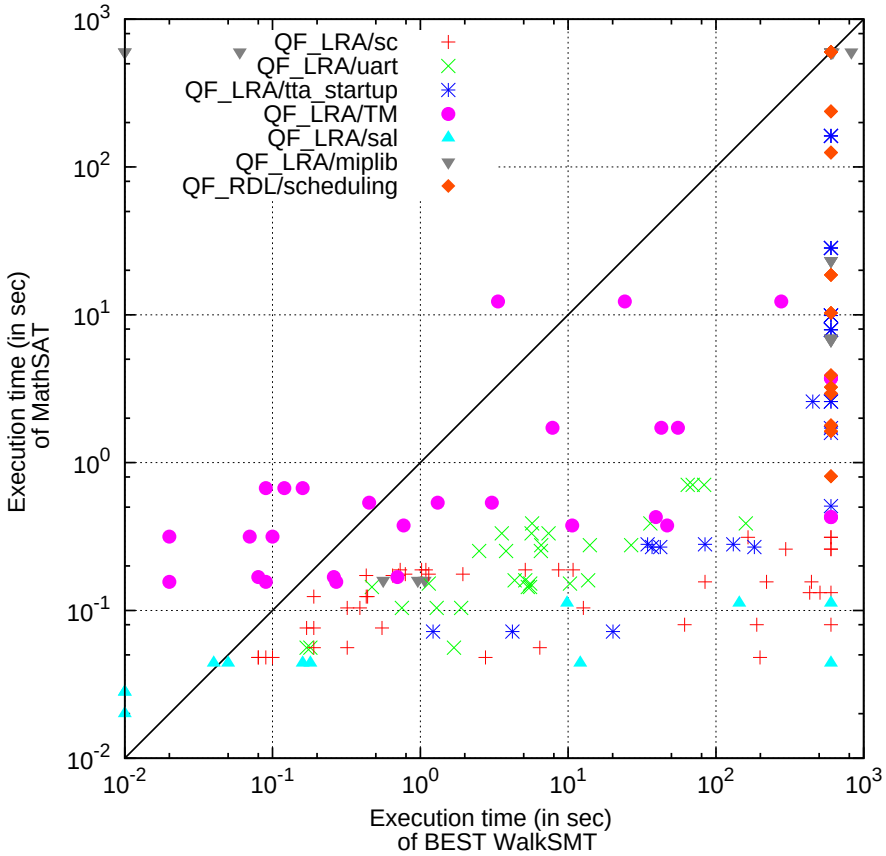
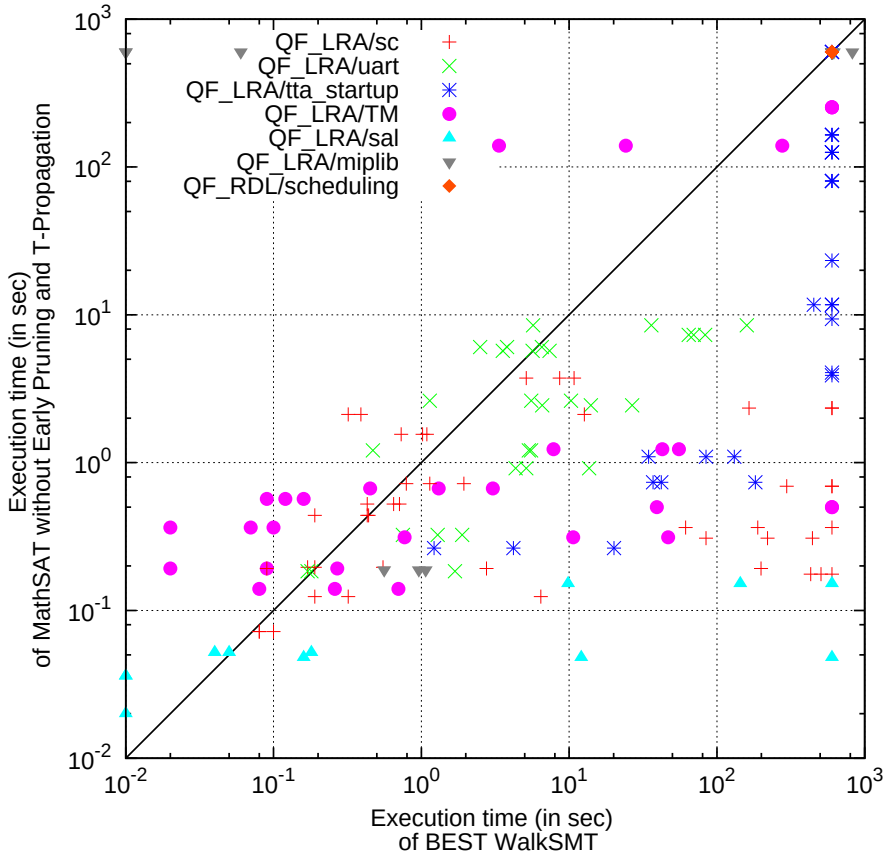
# Configurations of WalkSMT on SMT-LIB Instances



# WalkSMT vs MathSAT on SMT-LIB Instances



# Results on SMT-LIB Instances

- Learning the discovered $\mathcal{T}$-lemmas is crucial
- optimizations are very significant
- huge gap between WALKSMT and MathSAT:
  - early pruning and $\mathcal{T}$-propagation cannot be applied to WALKSMT since it works on complete truth assignment
  - DPLL-based SAT solvers outperform SLS-based ones on industrial, structured instances since Boolean Constraint Propagation is fully exploited

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# Randomly-generated Instances

3-CNF formula generated in terms of $\langle m, n, a \rangle$

- $m$ clauses
- $n$ $\mathcal{T}$-variables
- $a$ $\mathcal{T}$-atoms $\sum_{i=0}^{4} c_i x_i \leq c_0$ where
  - variable $x_i$ chosen with probability $1/n$
  - constant terms $c$ and $c_i$ randomly taken in $[-100, 100]$

## Plots

- plots represent the execution time vs ratio $r = m/a$
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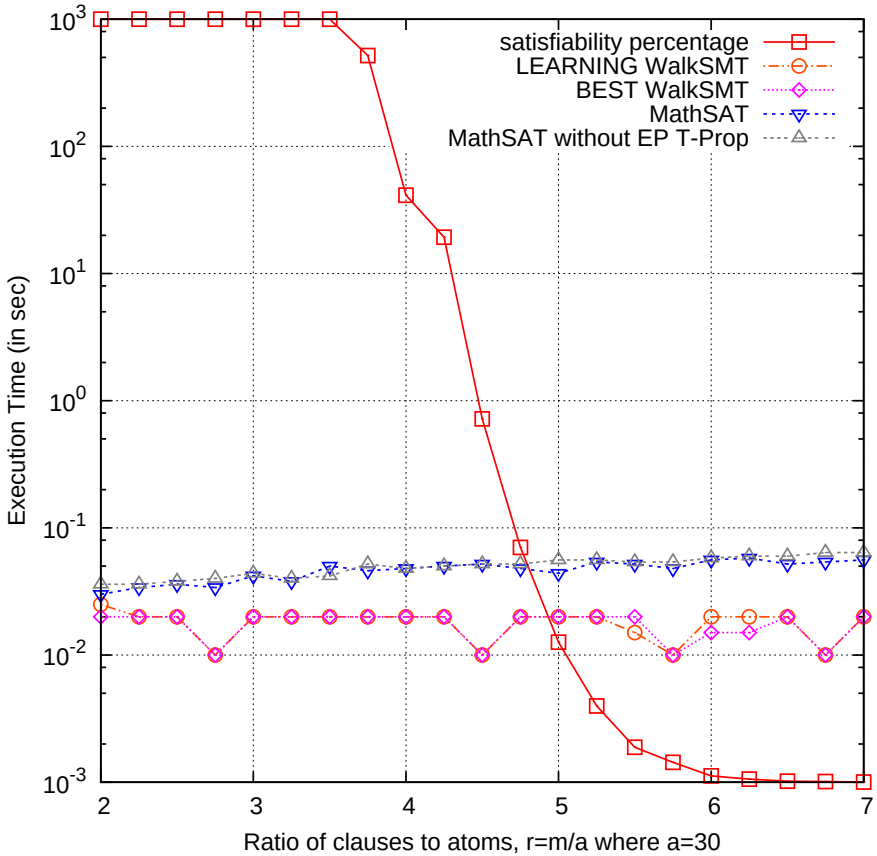


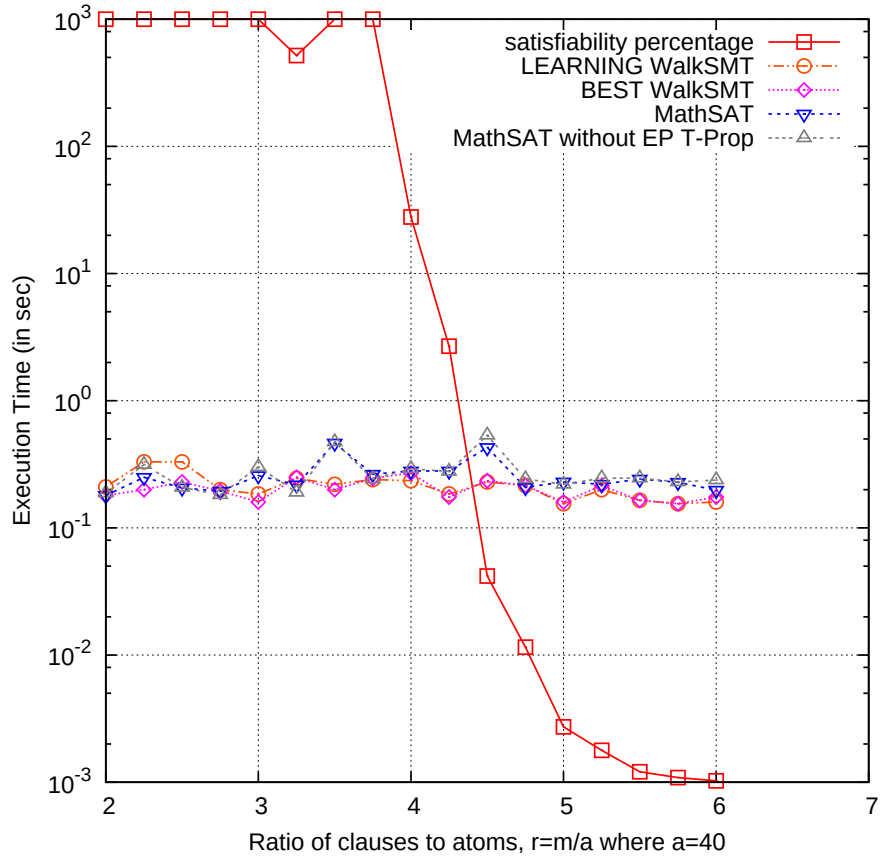

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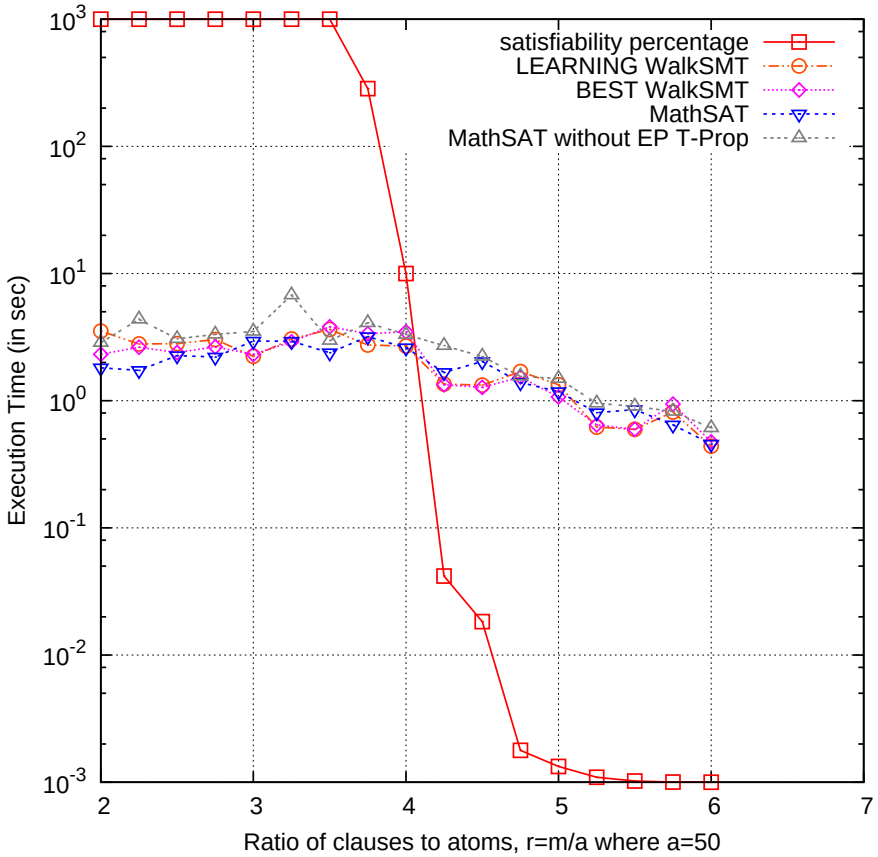


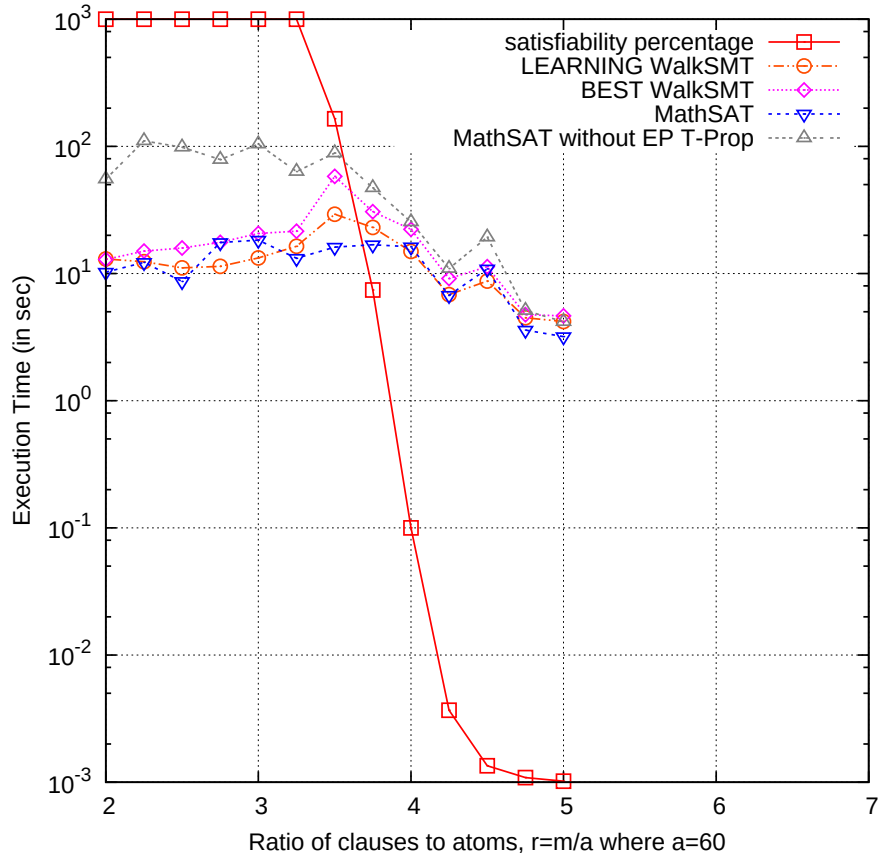

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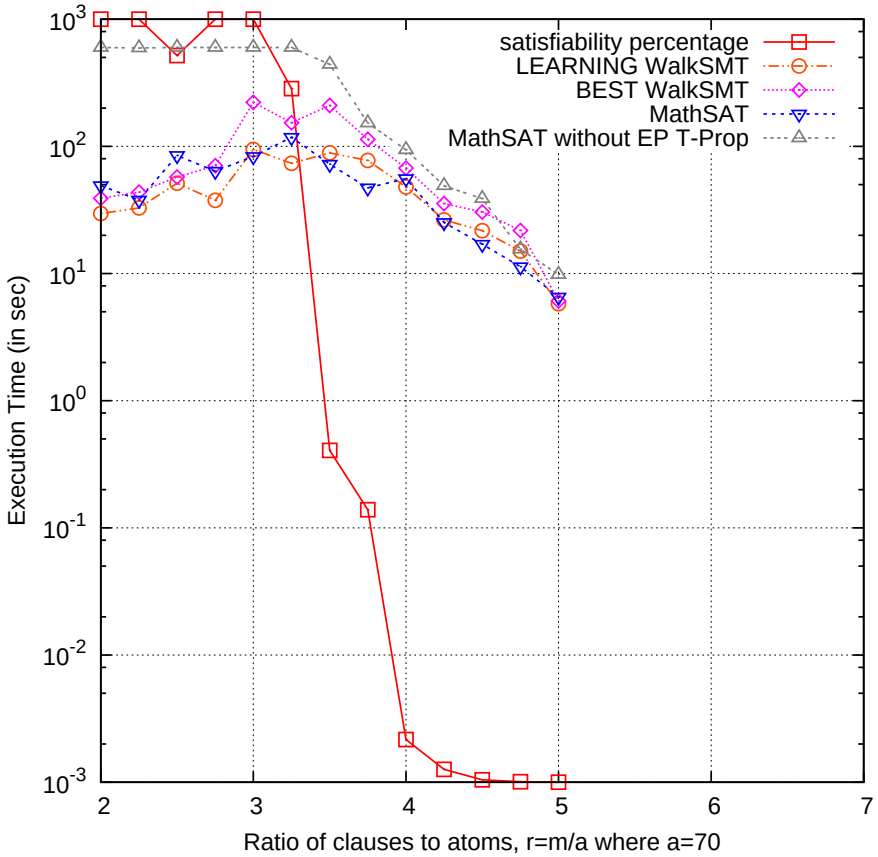


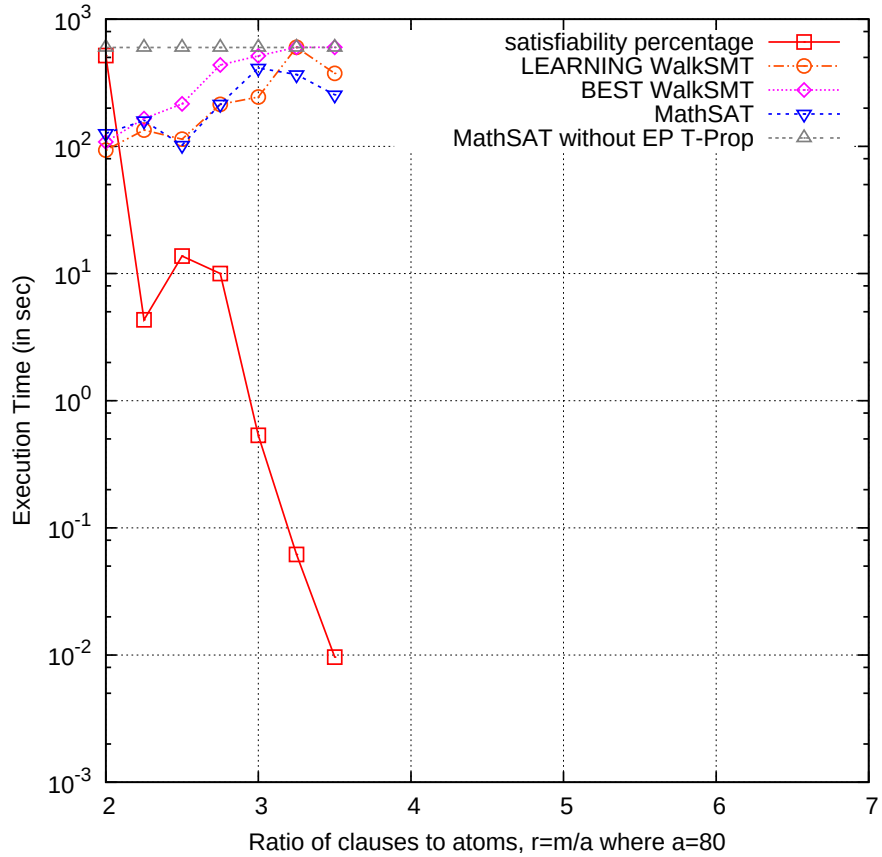

# Results on Random Instances

- multiple learning and filtering not very effective
- small difference between performance of WALKSMT and MathSAT

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- SLS for SMT: interesting research problem
- presented WALKSMT, a prototype SLS-based SMT procedure
- so far performances far from DPLL-based SMT solvers on SMT-LIB instances, comparable on random ones

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